# Without Looking Closer, it May Seem Cheap: Low Interest Rates and Government Borrowing* 

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January 9, 2019


#### Abstract

Are periods of low interest rates advantageous times for governments to increase expenditure by issuing debt? Because borrowing costs are lower, some economists have argued that the answer is yes. We argue that the logic used in reaching this conclusion may in fact be too simplistic. Whether or not it is a good time to issue debt depends not on whether interest rates are low, but rather on why interest rates are low. We show that if interest rates are low because of an increased preference for saving, then fiscal sustainability requires increasing debt in a period of low interest rates. In contrast, if interest rates are low because of a decline in trend output growth, then it is not sustainable to increase deficit financed spending.


JEL Classification: E43; H60.
Keywords: Interest Rates; Fiscal Sustainability.

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## 1 Introduction

With global real interest rates at historic lows, does it follow that now is a good time for governments to engage in fiscal expansion and increase debt? The logic behind arguments in the affirmative is quite simple - with interest rates low, it is a comparatively good time to borrow. In spite of this simple and seemingly unassailable logic, in this paper we argue that the case for increasing debt in a time of low interest rates is in fact not so clean-cut. In particular, when thinking about whether it is a "good" or "bad" time to increase debt, we argue that it matters not so much whether interest rates are low, but why interest rates are low.

For the purposes of this paper, we focus not on a measure of aggregate welfare or output, but rather on fiscal sustainability. Fiscal sustainability requires that the present discounted value of government debt in the far off future approaches zero. A sustainable fiscal plan requires that newly issued government debt equal the present discounted value of future primary surpluses, defined as the difference between tax revenues and non-interest expenses. We define a period to be a "good" time for an increase in government debt if the present discounted value of primary surpluses increases when interest rates fall (holding all future fiscal behavior constant), and a "bad" time to increase debt if the present discounted value of primary surpluses falls when interest rates fall.

In our exercises we consider two different exogenous reasons for why the real interest rate could be low - trend productivity growth is low or the subjective discount rate of households is low (i.e. households are relatively patient). We work through the logic in a two period endowment economy and in a standard RBC model which takes transition dynamics into account. In both economies, whether or not it is a good time for the government to increase its debt depends on the exogenous force driving interest rates down. ${ }^{1}$ While some previous work has observed that low growth can have important implications for fiscal sustainability, we are the first to demonstrate that these alternative sources of low interest rates can have differential effects on fiscal sustainability in both steady state and in transition dynamics. ${ }^{2}$ More than drawing policy prescriptions from our quantitative analysis, we show that the conclusions emerging from the basic RBC framework remain true in a richer framework under a standard and reasonable parameterization.

## 2 A Two Period Model

We begin with a stylized two-period endowment economy without uncertainty. A representative household has preferences over consumption, $C$. The household can save through one-period bonds, $B$, at a given real interest rate, $r$. The household is subject to a lump sum tax/transfer from a

[^1]government, $T$. Formally:
\[

$$
\begin{equation*}
\max _{C_{t}, B_{t}, C_{t+1}} \ln C_{t}+\beta \ln C_{t+1} \tag{1}
\end{equation*}
$$

\]

subject to

$$
\begin{aligned}
C_{t}+B_{t}-B_{t-1} & \leq Y_{t}-T_{t}+r_{t-1} B_{t-1} \\
C_{t+1} & \leq B_{t}\left(1+r_{t}\right)+Y_{t+1}-T_{t+1}
\end{aligned}
$$

where $Y$ represents income, $\beta \in(0,1)$ is the discount factor. The first order condition yields the standard Euler equation:

$$
\begin{equation*}
1+r_{t}=\frac{1}{\beta} \frac{C_{t+1}}{C_{t}} \tag{2}
\end{equation*}
$$

Output grows at an exogenous rate, $z_{t}$ :

$$
\begin{equation*}
Y_{t+1}=\left(1+z_{t}\right) Y_{t} \tag{3}
\end{equation*}
$$

The government enters period $t$ with a given stock of debt, $D_{t-1}$, consumes some output and finances it with a mix of lump sum taxation and debt issuance. Its period flow budget constraints are:

$$
G_{t+i}+r_{t+i-1} D_{t+i-1} \leq T_{t+i}+D_{t+i}-D_{t+i-1} \quad \text { for } i=0,1
$$

$G_{t+i}$ is government consumption.
Fiscal sustainability requires $D_{t+1}=0$ - i.e. that the government does not "die" in debt. Imposing this allows us to solve for new debt issued in $t$ as:

$$
\begin{equation*}
D_{t}=\frac{1}{1+r_{t}}\left[T_{t+1}-G_{t+1}\right] \tag{4}
\end{equation*}
$$

This expression requires that new debt issued in period $t$ equals the present discounted value of the future primary surplus. We assume the government consumes a known fraction of output in period $t+1, G_{t+1} / Y_{t+1}=g_{t+1} \in(0,1)$, and also that tax revenue is a fraction of output, $T_{t+1} / Y_{t+1}=\tau_{t+1}^{T} \in(0,1)$. These fractions need not be constant across periods. These definitions, plus (3), together allow us to rewrite (4) as:

$$
\begin{equation*}
\frac{D_{t}}{Y_{t}}=\frac{1+z_{t}}{1+r_{t}}\left[\tau_{t+1}^{T}-g_{t+1}\right] \tag{5}
\end{equation*}
$$

Market-clearing in the economy requires the household hold all debt issued by the government, which gives the usual resource constraint: $Y_{t}=C_{t}+G_{t}$. This implies that $C_{t}=\left(1-g_{t}\right) Y_{t}$. We can use this in conjunction with the Euler equation, (2), to solve for an expression for the equilibrium interest rate:

$$
\begin{equation*}
1+r_{t}=\frac{1}{\beta} \frac{1-g_{t+1}}{1-g_{t}}\left(1+z_{t}\right) \tag{6}
\end{equation*}
$$

Outside of a change in current or future government spending, from (6) we can deduce that the equilibrium interest rate can fall for one of two reasons: either there is an increased propensity to save (i.e. the household is relatively more patient, so $\beta$ is higher), or the growth rate of productivity between the present and the future, $z_{t}$, declines.

Equation (6) can be combined with (5) to derive an expression for the current debt-gdp ratio consistent with fiscal sustainability:

$$
\begin{equation*}
\frac{D_{t}}{Y_{t}}=\beta \frac{1-g_{t}}{1-g_{t+1}}\left(\tau_{t+1}^{T}-g_{t+1}\right) . \tag{7}
\end{equation*}
$$

The debt-gdp ratio consistent with fiscal sustainability depends only on the household's discount factor, $\beta$, and the tax and spending shares, $\tau^{T}$ and $g$, respectively. It does not depend on $z_{t}$. How does a low interest rate effect the affordability of increased debt? Assume that the government initially is running a deficit, so that the debt-to-gdp ratio and the future primary surplus are both positive. If the real interest rate falls because of an increase in $\beta$, then, given future tax and spending plans, the government should increase its debt-gdp ratio by either increasing period $t$ spending or cutting period $t$ taxes. In contrast, if the real interest rate falls because of a decline in $z_{t}$, then fiscal sustainability requires that the government not adjust its current debt-gdp ratio. In spite of a low interest rate, increasing debt in this case would necessitate either a future increase in taxes, a decrease in spending, or some combination thereof.

## 3 An Infinite Horizon RBC Model

In this section, we consider an infinite horizon real business cycle model with endogenous capital accumulation, variable labor supply, and different distortionary tax instruments. We show that the same basic intuition carries over to this more general framework.

### 3.1 The Model

An infinitely-lived representative household receives flow utility from consumption and disutility from labor. It can save by accumulating bonds or accumulating capital. Formally:

$$
\begin{gather*}
\max _{\left\{C_{t+j}, B_{t+j}, N_{t+j}, K_{t+j}\right\}} \quad \mathbb{E}_{t} \sum_{j=0}^{\infty} \beta^{t}\left(\ln C_{t+j}-\theta \frac{N_{t+j}^{1+\chi}}{1+\chi}\right)  \tag{8}\\
\text { s.t. } \\
C_{t+j}+I_{t+j}+B_{t+j}-B_{t+j-1} \leq\left(1-\tau^{N}\right) w_{t+j} N_{t+j}+\left(1-\tau^{K}\right) R_{t+j} K_{t+j}+\Pi_{t+j}+T_{t+j}+r_{t+j-1} B_{t+j-1},  \tag{9}\\
K_{t+j}=I_{t+j}+(1-\delta) K_{t+j-1}, \tag{10}
\end{gather*}
$$

(9) is the flow budget constraint and (10) is the law of motion for capital. $K_{t-1}$ and $B_{t-1}$ are the stocks of capital and bonds with which the household enters a period and are given. $C_{t}$ is consumption and $N_{t}$ is labor supply. The parameter $\chi \geq 0$ is the inverse Frisch labor supply elasticity
and $\theta$ is a scaling parameter. $I_{t}$ is investment. Capital depreciates at $\delta \in(0,1)$. The real wage is $w_{t}$ and the real rental rate on capital is $R_{t} . \tau^{N}$ and $\tau^{K}$ are fixed tax rates on labor and capital income, respectively. $\Pi_{t}$ is profit distributed from ownership of the firm and $T_{t}$ is a lump sum transfer positive or negative depending on whether is a tax or a subsidy - from the government.

The representative firm produces output with a standard Cobb-Douglas production function:

$$
\begin{equation*}
Y_{t}=K_{t}^{\alpha}\left(Z_{t} N_{t}\right)^{1-\alpha} \tag{11}
\end{equation*}
$$

$Z_{t}$ is exogenous productivity. Profit maximization by the firm implies $w_{t}=(1-\alpha) Y_{t} / N_{t}$ and $R_{t}=\alpha Y_{t} / K_{t}$. The firm earns no profit in equilibrium.

Government spending, $G_{t}$, and transfers, $T_{t}$, are assumed to be fixed fractions of output each period, $g \in(0,1)$ and $\tau^{T} \in(0,1)$, respectively. We write the present discounted values of government spending and of revenue as fractions of output recursively as:

$$
\begin{align*}
\frac{E X_{t}}{Y_{t}} & =\frac{1}{1+r_{t}} \mathbb{E}_{t} \frac{Y_{t+1}}{Y_{t}}\left(\frac{G_{t+1}}{Y_{t+1}}+\frac{T_{t+1}}{Y_{t+1}}+\frac{E X_{t+1}}{Y_{t+1}}\right)  \tag{12}\\
\frac{R E V_{t}}{Y_{t}} & =\frac{1}{1+r_{t}} \mathbb{E}_{t} \frac{Y_{t+1}}{Y_{t}}\left(\frac{\tau^{N} w_{t+1} N_{t+1}}{Y_{t+1}}+\frac{\tau^{K} R_{t+1} K_{t+1}}{Y_{t+1}}+\frac{R E V_{t+1}}{Y_{t+1}}\right) \tag{13}
\end{align*}
$$

We then define a variable we call fiscal sustainability, $S U S_{t}$, as the difference between these two:

$$
\begin{equation*}
S U S_{t}=\frac{R E V_{t}}{Y_{t}}-\frac{E X_{t}}{Y_{t}} \tag{14}
\end{equation*}
$$

Financial market-clearing requires that the household hold government debt. This gives rise to a standard aggregate resource constraint:

$$
\begin{equation*}
Y_{t}=C_{t}+I_{t}+G_{t} \tag{15}
\end{equation*}
$$

We assume that aggregate productivity obeys a non-stationary but deterministic process where $z \geq 0$ is (a constant) growth rate: $Z_{t+1}=(1+z) Z_{t}$.

Along the balanced growth path, output grows at a constant rate given by $\frac{Y_{t+1}}{Y_{t}}=1+z$. Similarly, consumption grows at the same rate as output, so $\frac{C_{t+1}}{C_{t}}=1+z$. This means that the steady state real interest rate can be written:

$$
\begin{equation*}
1+r^{*}=\frac{1+z}{\beta} \tag{16}
\end{equation*}
$$

Note that this is the same expression for the equilibrium real interest rate as in the two period model, (6). Given firm optimality conditions and the expression for the steady state real interest rate, the steady state values of the fiscal variables are:

$$
\begin{equation*}
\frac{E X^{*}}{Y^{*}}=\frac{\beta}{1-\beta}\left(g+\tau^{T}\right) \tag{17}
\end{equation*}
$$

$$
\begin{align*}
\frac{R E V^{*}}{Y^{*}} & =\frac{\beta}{1-\beta}\left[(1-\alpha) \tau^{N}+\alpha \tau^{K}\right]  \tag{18}\\
S U S^{*} & =\frac{\beta}{1-\beta}\left[(1-\alpha) \tau^{N}+\alpha \tau^{K}-\left(g+\tau^{T}\right)\right] . \tag{19}
\end{align*}
$$

As in the two period model of Section 2, the growth rate of productivity cancels from the steady state fiscal sustainability condition, (19).

### 3.2 Quantitative Analysis with Transition Dynamics

We now undertake a quantitative analysis to show that: $i$ ) the message of the paper holds true in a more realistic framework that is disciplined with a reasonable parameterization, and $i i$ ), that our main result also holds when taking transition dynamics into account. The unit of time is a quarter. We assume that $\beta=0.99$ and $\delta=0.025$. The inverse Frisch labor supply elasticity, $\chi$, is set to 1 . The parameter governing capital's share of income, $\alpha$, is set to $1 / 3$ and the parameter governing the disutility of labor, $\theta$, is set to be consistent with steady state labor hours of $1 / 3$. We assume that the steady state growth rate of productivity is 0.005 ; this translates into a two percent annualized rate of trend growth in output, and a steady state annualized real interest rate of about 6 percent. ${ }^{3}$ We assume that $g=0.2, \tau^{T}=0.1$, and $\tau^{N}=0.35$. We then solve for a value of $\tau^{K}$ to be consistent with $S U S^{*}=0.5$. This implies $\tau^{K}=0.22$.

We consider the following quantitative experiment. Up until period $t$, we assume that the economy sits in the balanced growth path. Then, in period $t$, there is an unexpected and permanent change in either $z$ or $\beta$ which lowers the steady state real interest rate by 100 basis points (i.e. from 6 percent annualized to 5 percent). We then trace out dynamic paths of variables as the economy transitions to the new balanced growth path.

Figure 1 plots time paths of $\log$ output and the real interest rate in response to one of these changes. The blue lines show the time paths if the economy were to stay in the original balanced growth path. The dashed red lines show the transition dynamics when either $z$ or $\beta$ change. The left column considers a reduction in $z$, while the right column considers an increase in $\beta$. The upper row plots the time paths of log output, while the bottom row plots the time paths of the real interest rate.

The reduction in $z$ and increase in $\beta$ lead to the same change in the steady state real interest rate and produce similar transition dynamics. In both cases, the real interest rate initially increases slightly before steadily declining towards the new steady state while the stock of capital adjusts. The reduction in $z$ initially leads to a small output increase (owing to a negative wealth effect on labor supply), but thereafter output grows at a slower rate forever. In the case of an increase in $\beta$, output initially increases and remains on a higher level trajectory than it otherwise would have, though the change in $\beta$ does not affect the long run growth rate of output.

[^2]

Figure 1: Time Paths of Output and Interest Rate
Notes: this figure plots the paths of output (top row) and interest rates (bottom row) in response to exogenous shocks to productivity (left column) and household patience (right column). Solid lines display variable paths in the absence of the corresponding shock and dashed lines display paths with the shock.


Figure 2: Time Paths of Fiscal Sustainability
Notes: this figure plots the path of future primary surpluses in response to exogenous shocks to productivity declines (left column) and household patience (right column). Solid lines display variable paths in the absence of the corresponding shock and dashed lines display paths with the shock.

Figure 2 shows the time paths of the fiscal sustainability variable, $S U S_{t}$, conditional on permanent changes in either $z$ or $\beta$. Consistent with the analytical analysis above in Section 3.1, the steady state value of $S U S_{t}$ is unaffected by the decline in $z$ while it increases after an increase in $\beta$. In
terms of transition dynamics, initially the value of $S U S_{t}$ temporarily falls after the reduction in $z$. Intuitively, this initial decline occurs because expected output growth falls immediately, whereas it takes some time for the real interest rate to decline. In contrast, the value of $S U S_{t}$ initially jumps up after a permanent increase in $\beta$. It initially undershoots relative to its new steady state. This undershooting also occurs because it takes time for the real interest rate to decline.

Figure 2 makes clear in a quantitative framework the main point from Section 2: whether or not it is sustainable to issue more debt in a period of low interest rates depends on why interest rates are low. When low interest rates are driven by a decline in trend productivity growth, maintaining sustainability requires that the debt-to-gdp ratio go down in the short run and not change in the long run. The opposite is the case when low interest rates are caused by an increase in patience. ${ }^{4}$

## 4 Conclusion

We have argued that the seemingly unassailable argument that governments ought to issue more debt when interest rates are low is in fact a bit too simple. Rather, why interest rates are low is what matters for the affordability of public debt, not whether interest rates are low. In doing so, we demonstrate a simple - but critical - point that not all low interest rate scenarios are created equal and, accordingly, policy makers should take seriously the endogenous reasons driving the behavior of interest rates. Put differently, unconditional statements about the desirability of borrowing in low interest environments may be misleading.

[^3]
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[^0]:    ${ }^{*}$ We are particularly grateful to Rüdiger Bachmann, Bill Lastrapes, Nam Vu, and seminar participants at Miami University and at the Fall 2017 Midwest Macro Meeting. We are also appreciative of comments and suggestions from the Editor, Pierre-Daniel Sarte, and an anonymous referee. The usual disclaimer applies. Corresponding author: Julio Garín. Author contact information: Julio Garín - jgarin@cmc.edu, Robert Day School of Economics and Finance, Bauer Center North, 500 E. 9th Street, Claremont, CA 91711, USA; Robert Lester - rblester@colby.edu, 5240 Mayflower Hill, Waterville, ME 04901, USA; Eric Sims - esims1@nd.edu, 3036 Nanovic Hall, Notre Dame, IN 46637, USA; Jonathan Wolff - wolffjs@miamioh.edu, 3037 Farmer Business School, Miami University of Ohio, Oxford, OH 45056, USA.

[^1]:    ${ }^{1}$ In this paper we do no take a stand on why interest rates around the globe are currently low. Several scholars have addressed this question; for instance Fernald and Jones (2014), Gordon (2016), Laubach and Williams (2016), Laubach and Williams (2016), Carvalho, Ferrero, and Nechio (2016), Gagnon, Johannsen, and Lopez-Salido (2016), Hall (2016), and Negro, Giannone, Giannoni, and Tambalotti (2017), to name a few. These different explanations are captured in our model in an admittedly reduced form way through the subjective discount rate of households.
    ${ }^{2}$ Mehrotra (2017), building on insights in Ball et al. (1998), makes a similar point in a quantitative OLG model.

[^2]:    ${ }^{3}$ While this value is above the long-run real interest rates observed in the post-war period, because the steady state interest rate and the steady state growth rate jointly determine that parameter $\beta$, the steady state interest rate is important only to the extent that a $1 \%$ decline in the interest rate still generates a value of $\beta$ strictly less than unity. This can be seen in Equation (16).

[^3]:    ${ }^{4}$ These results are robust to changes in the parameter values and when government spending is productive and are available upon request.

