Notes 5: Linear Combination of Random Variables

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Introduction

- So far we focus on one variable at a time.
  - Examples.
- But, usually, we are interested in examining two or more r.v. together. Why?
- We need to use our measures of 'co-movement'.
  - Examples?

Introduction

- Sometimes we worry about a weighted average of two or more r.v.
  - Examples.
  - These examples will be function of the individual series, *plus* an extra term...
- As before, what we would like to know?
- Recall some definitions:
  - Expected Value of X.
  - Variance of X.

Measures of Dependence

- Suppose that we are interested in the probability that two events occur.
  - Independence and measure of dependence.
  - Covariance.
    - What we expect from its values?
    - How to estimate it?
- Covariance and independence.
- A word of caution about non-zero covariance.

Multivariate Probability Distributions

- Conditional and marginal probability distributions.
  - Conditional expectations.
  - Expected value.
  - Variance.
- Recall our well known transformation $Y = \beta_0 + \beta_1 X$.
  - Expected value.
  - Variance.
  - Covariance
    - Covariance between transformations of random variables.
- Problem with covariance.
  - How to solve it?
- Correlation coefficient.

Linear Combination of Two Random Variables

- Suppose X and Y are r.v. and we have

$$Z = \alpha + \beta_1 X + \beta_2 Y$$

  where α, β_1, and β_2 are constants.

- We are interested in:
  - Expected value of Z, $\mathbb{E}(Z)$.
  - Variance of Z, $\text{Var}(Z) =$
    1. Add in the definition of Z.
    2. Expand the quadratic form.

Example I: SAT Scores

We are interested in the moments of the Total (T) score from the SAT which is the Math score (M) plus the Verbal score (V). We have that

$$\mu_M = 520, \quad \sigma_M^2 = 120, \quad \sigma_M = 10.95$$

$$\mu_V = 480, \quad \sigma_V^2 = 143, \quad \sigma_M = 11.95$$

and $\text{Cov}(M, V) = 66$.

1. What is the expected value of T?
2. What is the variance of T?
3. What is the correlation coefficient between M and V?

Example II: SAT Prep Course

Let T_B be the total SAT before the prep course and let T_A the total SAT score after the test prep. From past data we have that

$$\mu_B = 1,000, \quad \sigma_B^2 = 395, \quad \sigma_B = 19.87$$

$$\mu_A = 1,111, \quad \sigma_A^2 = 399, \quad \sigma_A = 11.97$$

and $\text{Cov}(T_B, T_A) = 280$.

1. What is the distribution for the gain in SAT scores after taking a prep test?

Example III: Portfolio Returns

Let s be the returns on stocks and let b the returns on bonds. Assume that the agent has 75% of his portfolio on stocks and 25% on bonds. We also know that

$$\mu_s = 12\%, \quad \sigma_s^2 = 16.12, \quad \sigma_s = 4.01$$

$$\mu_b = 6\%, \quad \sigma_b^2 = 4.49, \quad \sigma_b = 2.12$$

and $\rho_{s,b} = 0.26$.

1. What is the expected value of the portfolio return, r?
2. What is the variance of r?

Example IV: Admission to Kentucky State University

The admissions index was established to quantify an assessment of a student's high school activities and ACT assessment. The admissions index is a numerical score calculated by multiplying the ACT by 10, the grade-point average by 100, and by adding the two sums. The equation is as follows:

$$\text{Index} = ACT \times 10 + GPA \times 100$$

Kentucky State University requires students to meet an admission index of 430 in order to be admitted unconditionally to the University. Let $A = ACT$ and $G = GPA$, assume further that

$$\mu_A = 20\%, \quad \sigma_A = 5.6$$

$$\mu_G = 2.75\%, \quad \sigma_G = 1.2$$

and $\rho_{A,G} = 0.45$.

1. What are the mean and the variance of the index, I?
2. Suppose the index, I, is normally distributed with the mean and variance found in the previous part. What fraction of applicants are eligible for enrollment to Kentucky State?

Material Covered So Far

Review.