Economics 30330: Statistics for Economics
Problem Set 5
University of Notre Dame
Instructor: Julio Garín
Spring 2012

Due Date: Beginning of class on Monday, March 26th. Please complete the assignment in the allotted space. You may work in groups, but you need to turn in your own work.

Continuous Probability Distributions (120 Points)

1. A management consulting firm conducted a study of service time at the drive-up window at McDonalds. They found that the average time between placing an order and receiving the order was 2.78 minutes.

   (a) What is the variance of the waiting time?

   (b) What is the probability that a customer's waiting time is more than 5 minutes?

   (c) What is the probability that a customer's waiting time is less than 2.78 minutes?

2. Eric's weekly expenditures at North Dining Hall are normally distributed with a mean of \$80 and a standard deviation of \$10. Eric wants to budget an amount each week he can spend at the cafeteria. How much should Eric budget if he wants to spend less than the budgeted amount 90 percent of the time?

3. Let $z \sim \mathcal{N}(0,1)$. Calculate,

   (a) $P(-1 \leq z \leq 0)$.

   (b) $P(-2.51 \leq z \leq 0)$.

   (c) $P(-1.75 \leq z \leq -1.04)$.

   (d) Find z^* such that $P(z \geq z^*) = 0.6915$

4. Your classmates Luke and Shawn are decent golfers but they are always bragging about their ability to hit the monster drive. Just last week, Shawn claimed that his drives routinely go 295 yards. The average drive for a professional player on the PGA tour travels 272 yards with a standard deviation of 8 yards.

   (a) If the distance of a professional drive is normally distributed, what fraction of drives exceed 295 yards? What can you said about Luke and Shawn's statements?

   (b) Again, assuming normality, how long would a drive have to be in the top 5 percent of drives hit on the professional tour?

5. Delta airlines quotes a flight time of 2 hours, 5 minutes for its flights from Cincinnati to Tampa. Suppose we believe that actual flight times are uniformly distributed between 2 and 2.333 hours (2 hours 20 minutes). Let X be the time flight for this route.

   (a) Graph the PDF of X.

   (b) What is the probability that the flight will be no more than 5 minutes late?

   (c) What is the expected flight time and what is the standard deviation of flight times?

6. Most publications that describe colleges and universities report the 25th and 75th percentile of SAT scores for the entering students. Suppose that the SAT scores for entering students at ND are normally distributed with a mean of 1440 and a standard deviation of 120.

   (a) What are the 25th and 75th percentile SAT scores?

   (b) The 25th and 75th percentile of SAT scores for incoming freshman at UVA are 1220 and 1440, respectively. If SAT scores for these students are normally distributed, what is the mean and standard deviation of SAT scores at UVA?

7. The proportion of time Y that an industrial robot is in operation during a 40-hour week is a random variable with probability density function

$$f(y) = \begin{cases} 2y & \text{for } 0 \leq y \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find $\mathbb{E}(Y)$ and $\text{Var}(Y)$.

(b) For the robot under study, the profit X for a week is given by $X = 200Y - 60$. Find $\mathbb{E}(X)$ and $\text{Var}(X)$.

(c) Find an interval in which the profit should lie for at least 75% of the weeks that the robot is in use.

8. The SAT and ACT college entrance exams are taken by thousands of students each year. The mathematics portions of each of these exams produce scores that are approximately normally distributed. In recent years SAT mathematics exam scores have averaged 480 with standard deviation 100. The average and standard deviation for ACT mathematics scores are 18 and 6, respectively.

(a) An engineering school sets 550 as the minimum SAT math score for new students. What percentage of students will score below 550 in a typical year?

(b) What score should the engineering school set as a comparable standard on the ACT math test?

9. As a measure of intelligence, mice are timed when going through a maze to reach a reward of food. The time (in seconds) required for any mouse is a random variable Y with a density function given by

$$f(y) = \begin{cases} \frac{b}{y^2} & y \geq b \\ 0 & \text{elsewhere} \end{cases}$$

where b is the minimum possible time needed to traverse the maze

(a) Show that $f(y)$ has the properties of a denstiy function.

(b) Find $F(y)$.

(c) Find $P(Y > b + c)$ for a positive constant c.

(d) If c and d are both positive constants such that $d > c$, find $P(Y > b + d|Y > b + c)$.

10. Suppose that X has a density function

$$f(y) = \begin{cases} kx(1-x) & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find the value of k that makes $f(x)$ a probability density function.

(b) Find $P(0.4 \leq X \leq 1)$.

(c) Find $P(X < 0.4 | X \leq 0.8)$.

(d) Find the 95th percentile of X.

(e) Find a value of x_0 so that $P(X < x_0) = 0.95$.

(f) Compare the values obtained in part $d) - e)$. Explain the relationship between these two values.

11. Show that the maximum value of the normal density with parameters μ and σ is $1/(\sigma\sqrt{2\pi})$ and occurs when $x = \mu$. *Hint:* You are just maximizing a function.

12. Explain intuitively:

    (a) What happens to the first and second moments of a variable when every observation its doubled?

    (b) What happens to the first and second moments of a variable when we add a 3 to each observation?

    (c) What happens to the first and second moments of a variable when, after multiplying each observation by two, we add a 3 to each value?