

Economics 30330: Statistics for Economics
Problem Set 3
University of Notre Dame
Instructor: Julio Garín
Spring 2012

Due Date: Beginning of class on Wednesday, February 15th. Please complete the assignment in the allotted space. You may work in groups, but you need to turn in your own work.

Basics of Probability (100 points)

1. Bruno's offers pizzas with two different sizes, three different types of crusts, and up to 10 toppings. How many different types of pizzas can be made?

2. Suppose that two runners from team A and three runners from team B participate in a race. If all five runners have equal ability and there are no ties, what is the probability that two runners from team A will finish first and second, and that the three runners from team B will finish third, fourth, and fifth?

3. A high school senior applies for admission to the University of Northern Detroit and University of Southern Colorado. She estimates the probability of acceptance at ND at 0.8, the probability of acceptance at USC at 0.3, and the probability of being admitted to both at 0.2. What is the chance she will not be accepted at either school?

4. Consider the experiment of rolling a pair of dice.
 - (a) How many sample points are possible?

(b) List the sample points.

(c) What is the probability of rolling a 7?

(d) What is the probability of rolling a 9 or greater?

(e) What is the probability of rolling an odd number?

(f) What is the probability of rolling an even number?

5. Listed below is a two-by-two table. In period 1, events A or B can happen. In period 2, outcome C or D will result. Assume $\Pr(C|B) = 0.15$ and $\Pr(D|A) = 0.7$. Please fill in the missing boxes below.

	Outcome A	Outcome B	
Outcome C		0.03	
Outcome D			

6. Small cars may not be as safe as large cars. Small cars accounted for 18% of all vehicles on the road. Accidents involving small cars led to 11,898 fatalities during a recent year. Assume the probability that a small car is involved in an accident is 0.18, the probability that an accident involving a small car leads to a fatality is 0.128. Similarly, the probability that the accident leads to a fatality driving a car that is not small is 5%. You hear of an accident involving a fatality. Assume that the probability of getting into an accident is independent of car size. What is the probability that a small car was involved?

7. The prior probabilities for events A_1 , A_2 and A_3 are $\Pr(A_1) = 0.2$, $\Pr(A_2) = 0.5$, and $\Pr(A_3) = 0.3$. The conditional probabilities of B given A_1 , A_2 and A_3 are $\Pr(B|A_1) = 0.5$, $\Pr(B|A_2) = 0.4$, and $\Pr(B|A_3) = 0.3$.

(a) Find $\Pr(B \cap A_1)$, $\Pr(B \cap A_2)$ and $\Pr(B \cap A_3)$.

(b) Apply Bayes' theorem to compute the posterior probability $\Pr(A_2|B)$.

(c) Use the tabular approach to applying Bayes' theorem to compute $\Pr(A_1|B)$, $\Pr(A_2|B)$, and $\Pr(A_3|B)$.

8. According to the Arizona Chapter of the American Lung Association, 7% of the population has lung disease. Of those having lung disease, 90% are smokers; of those not having lung disease, 25.3% are smokers. What is the probability that a randomly selected smoker has lung disease?

9. Prove the following:

(a) The mean and variance of the z-score are 0 and 1, respectively.

(b)
$$\sum_{i=1}^n (x_i - \bar{x}) = 0$$

10. Recall the formula for population variance when completing the following exercises.

(a) Show that the sum of squared deviations can be written as $\sum_{i=1}^N x_i^2 - N\bar{x}^2$.

(b) Show that the variance can be written as $\sigma^2 = \frac{1}{N} \sum_{i=1}^N x_i^2 - \mu^2$.

(c) Show that the variance can also be written as $\sigma^2 = \frac{1}{N} \sum_{i=1}^N x_i(x_i - \mu)$.