Economics 30330: Statistics for Economics
Problem Set 0 - Suggested Answers
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Review of Summations (40 points)

1. Evaluate:

   (a) $\sum_{j=0}^{3} 2^j = 2^0 + 2^1 + 2^2 + 2^3 = 15$

   (b) $\sum_{j=0}^{3} j^2 = 0^2 + 1^2 + 2^2 + 3^2 = 14$

   (c) $\sum_{j=1}^{5}(2j-3) = 2\sum_{j=1}^{5} j - 3\sum_{j=1}^{5} = 2(1+2+3+4+5) - (3)(5) = 15$

   (d) $\sum_{j=1}^{1000} 5 = 5\sum_{j=1}^{1000} = 5000$

2. Show that:

   (a) $\dfrac{\sum_i (X_i + Y_i) + \sum_i X_i - \sum_i Y_i}{\sum_i X_i} = 2$

   $$\begin{aligned}\frac{\sum_i (X_i + Y_i) + \sum_i X_i - \sum_i Y_i}{\sum_i X_i} &= \frac{\sum_i (X_i + Y_i + X_i - Y_i)}{\sum X_i} \\ &= \frac{\sum_i 2X_i}{\sum_i X_i} \\ &= 2\frac{\sum_i X_i}{\sum_i X_i} \\ &= 2 \quad \blacksquare\end{aligned}$$

   (b) $\dfrac{\sum_i (X_i^2 + 2X_iY_i + Y_i^2) - \sum_i (X_i^2 - 2X_iY_i + Y_i^2)}{\sum_i 8X_iY_i} = \dfrac{1}{2}$

   $$\begin{aligned}\frac{\sum_i (X_i^2 + 2X_iY_i + Y_i^2) - \sum_i (X_i^2 - 2X_iY_i + Y_i^2)}{\sum_i 8X_iY_i} &= \frac{\sum_i (X_i^2 + 2X_iY_i + Y_i^2 - X_i^2 + 2X_iY_i - Y_i^2)}{\sum_i 8X_iY_i} \\ &= \frac{\sum_i 4X_iY_i}{\sum_i 8X_iY_i} \\ &= \frac{4}{8}\frac{\sum_i X_iY_i}{\sum_i X_iY_i} \\ &= \frac{1}{2} \quad \blacksquare\end{aligned}$$

3. Calculate/expand the following:

   (a) $\sum_{i=1}^{4} i^2 = 1^2 + 2^2 + 3^2 + 4^2 = 30$

   (b) $\sum_{i=1}^{5} x_i^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2$

   (c) $\sum_{i=1}^{3} x = x + x + x = 3x$

   (d) $\sum_{i=1}^{n} x^2 = x^2 + x^2 + ... + x^2 = nx^2$

(e) $\sum_{i=1}^{n} \frac{x}{n} = \frac{x + x + ... + x}{n} = \frac{nx}{n} = x$

4. Explain the difference between $\sum_i x_i^2$ and $(\sum_i x_i)^2$. Construct an example to show that, in general, these quantities are not equal.
   The expression $\sum_i x_i^2$ represents the sum of the squares of the data; to calculate the sum of squares, we square each observation and add up all those values. The expression $(\sum_i x_i)^2$ represents the square of the sum of the data; to calculate the squared sum, we add up all of the observations and square that value.

   One example:

   Let $x = [1, 5, 2]$, that is, $x_1 = 3, x_2 = 5, x_3 = 2$. Therefore,

$$\sum_{i=1}^{3} x_i^2 = 9 + 25 + 4 = 38$$

$$\left(\sum_{i=1}^{3} x_i\right)^2 = (3 + 5 + 2)^2 = 100$$