Notes V - Heterogeneous Agents & Incomplete Markets

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Motivation

- Why heterogeneity may be important:
  1. Aggregation bias.
  2. Heterogeneity in decisions and its impact on aggregates.

So Far...

- We have already analyzed heterogeneity:
  - Equilibrium endowment economy with a complete set of state-contingent securities.
  - Also, in some of the extensions of the RBC model.
  - Heterogeneity "didn't matter" since idiosyncratic shocks can be insured away.
    - Wealth distribution has no impact on aggregate dynamics.
- These models are useful but we cannot study distributional issues.
  - In this sense, complete markets are at odds with the data.
    - We will have to depart from them.

These Lectures

- We will assume that there is not a complete set of stage-contingent securities.
  - Incomplete-markets model.
- With aggregate shocks, this leads to important computational difficulties.
- To simplify, we will consider:
  - Only idiosyncratic shocks with exogenous restrictions on available assets.
- Agents will be *ex ante* homogenous.
  - But they cannot insure away all idiosyncratic risk.
    - They are *ex post* heterogeneous.

Why is This Difficult?

- In the models so far the Welfare theorems apply.
  - First best could be achieved by a competitive equilibrium.
    - Solving the Social Planner allocation was enough.
- With incomplete markets this is not the case.
  - We need to solve the competitive equilibrium.
    - This, among other things, requires solving for prices.
  - Dynamics and uncertainty will be different for individuals and the aggregate economy.
- With heterogeneous agents, the distribution of asset holdings is non-degenerate.
  - It becomes a state variable.
    - And it is an infinite object.

Preliminaries: Self-Insurance

Precautionary Motive and Buffer Stock

- With quadratic preferences nothing “bad” happens when consumption goes very low.
- With non-quadratic preferences, we will have a relevant third moment:
  - $U''' > 0$ known as “prudence”.
    - Curvature of the marginal utility function.
- Prudence is a motive for additional savings:
  - *Precautionary motive* or *self-insure*.
    - Consumption is expected to grow even if $\rho = r$.
  - If $\rho > r$ agents accumulate a “buffer stock” until precautionary motive is dominated by the agent’s relative impatience.
    - Why would we like to focus on this case?

Borrowing Constraints

- Two types of borrowing limits:
  1. *Natural borrowing constraint*.
     - The constraint self-imposed by the agent.
  2. *Ad-hoc*.
- Ad-hoc constraints are more stringent.
  - With prudence, a natural borrowing constraint will never bind.
- Ad hoc borrowing constraints will change our previous result:
  - There will be precautionary savings with quadratic utility.

Borrowing Constraints and Quadratic Utility

- Household can buy (or borrow, by selling) a riskless asset, a, that has a real interest r.
- Household's problem is:

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to:

$$c_t + a_{t+1} = (1+r)a_t + y_t$$
$$a_t \geq \phi$$

- What is c_t equal to with quadratic preferences and $\phi = 0$?

Result:

Even in the absence of prudence, in presence of borrowing constraints a rise in future income uncertainty leads to a rise in current savings and decline in current consumption.

Inspecting the Mechanism

- In the previous problem define "cash in hand", x_t, as

$$x_t = (1+r)a_t + y_t$$

- Assuming y_t is i.i.d., we can then write the problem as:

$$V(x_t) = \max_{c_t, x_{t+1}} u(c_t) + \frac{1}{1+\rho} \mathbb{E}_t V(x_{t+1})$$

subject to:

$$x_{t+1} = (1+r)(x_t - c_t) + y_{t+1}$$
$$x_t - c_t \geq \phi$$

- The borrowing constraint adds convexity to the problem.
  - The buffer stock saving with quadratic utility is driven by the ad hoc constraint.

Heterogeneous Agents Models

Bewley-Aiyagari Models

- Earlier models:
  - Bewley (1977).
  - Imrohoroğlu (1989).
- GE:
  - Huggett (1993).
  - Aiyagari (1994).
  - Krussell and Smith (1998).

Heterogeneous Agents Models

- We get away from the representative agent model.
  - We will assume incomplete markets.
    - To simplify: we rule out state-contingent securities completely.
- We will cover two versions of Bewley models:
  1. Huggett (1993): Endowment economy with bonds in zero net supply.
  2. Aiyagari (1994): Production economy.

Households' Problem

- There is a unit measure of infinitely-lived households solving:

$$\max_{\{c_t, a_{t+1} \in \mathcal{A}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} u(c_t)$$

subject to:

$$c_t + a_{t+1} = (1+r)a_t + w \cdot s_t$$
$$a_0 \text{ given}$$

- s_t can be interpreted as the worker's labor effort.
  - m-state Markov process with values given by vector $\bar{s}$ and transition matrix $\mathcal{P}$.
    - Model's source of uncertainty.
- Notice that a_{t+1} is constrained to be on a grid.
  - There is a borrowing limit.

Back to Aiyagari's.

Bellman Equation

$$V(a_h, \bar{s}_i) = \max_{a' \in \mathcal{A}} u\left[(1+r)a_h + w\bar{s}_i - a'\right] + \beta \sum_{j=1}^{m} \mathcal{P}(i,j) V(a', \bar{s}_j)$$

- Solving the Bellman generates a decision rule: $a' = g(a, s)$

Wealth-Employment Distributions

- Define unconditional distribution of (a_t, s_t) pairs:

$$\lambda_t(a, s) = \Pr(a_t = a, s_t = s)$$

  - With our assumptions, this gives us the distribution of agents over the state space.

- The law of motion for this distribution is:

$$\begin{aligned}
\lambda_{t+1}(a', s') &= \sum_{j=1}^{m} \sum_{i=1}^{n} \Pr(a_t = a_i, s_t = \bar{s}_j) \cdot \Pr(s_{t+1} = s' | s_t = \bar{s}_j) \\
&\quad \cdot \Pr(a_{t+1} = a' | a_t = a_i, s_t = \bar{s}_j) \\
&= \sum_{j=1}^{m} \sum_{i=1}^{n} \lambda_t(a_i, \bar{s}_j) \cdot \Pr(s_{t+1} = s' | s_t = \bar{s}_j) \cdot \mathcal{I}(a' | a_i, \bar{s}_j) \\
&= \sum_{j=1}^{m} \sum_{\{i : a' = g(a_i, \bar{s}_j)\}} \lambda_t(a_i, \bar{s}_j) \cdot \Pr(s_{t+1} = s' | s_t = \bar{s}_j)
\end{aligned}$$

- The law of motion depends on:
  1. $g(a, s)$.
  2. $\mathcal{P}$.
- We can calculate an invariant distribution.
- About λ.
  - The cross-section distribution of agents remains constant.
  - For a given r the population mean

$$\mathbb{E}(a)(r) = \sum_{a,s} \lambda(a, s) g(a, s)$$

    Can be interpreted in two ways:
    1. Across-time.
    2. Across households.

Huggett's "Pure Credit" Model

- Huggett (1993).
- Pure exchange economy.
  - Model is closed out by assuming that the bonds are in zero net supply.
- There is a lower bound on assets given by $a_i = -\phi$.
- Interest rate, r, will adjust to equate borrowing and lending.
- Wages are still set exogenously.

Equilibrium in the Huggett Model

Definition (Stationary Equilibrium)

Given ϕ, a stationary equilibrium is an interest rate r, a policy function $g(a,s)$, and a stationary distribution $\lambda(a,s)$ for which

1. *The policy function $g(a,s)$ solves the household's optimization problem;*
2. *The stationary distribution $\lambda(a,s)$ is induced by the $(\mathcal{P},\bar{s})$ and $g(a,s)$, satisfying*

$$\lambda(a',s') = \sum_{j=1}^{m} \sum_{\{i:a'=g(a_i,\bar{s}_j)\}} Prob(s'|s=\bar{s}_j)\cdot\lambda(a_i,\bar{s}_j)$$

3. *The loan market clears:*

$$\sum_{a,s} \lambda(a,s)g(a,s) = 0$$

Huggett's Equilibrium Computation

1. Fix $r = r_j$ for $j = 0$.
2. Solve household's problem for $g_j(a, s)$.
3. Calculate associated $\lambda_j(a, s)$.
4. Compute
$$\sum_{a,s} \lambda(a, s) g(a, s) = e_j^*$$
5. If $e_j^* > 0$, lower r.
6. Forward to $j + 1$ and repeat until $e_j^* = 0$

Aiyagari's Production Economy

- Aiyagari (1994).
- Households now save by buying capital.
  - Call it k instead of a.
- Representative firm pays:
  - $\tilde{r}$ for the rental rate of capital.
  - w for the households' labor effort.
- Capital depreciates at δ.
  - Could be mapped into baseline model. Baseline

Equilibrium in the Aiyagari Model

Definition (Stationary Equilibrium)

A stationary equilibrium is a decision rule $g(k,s)$, a probability distribution $\lambda(k,s)$, a rental rate $\tilde{r}$, a wage w, a capital stock K, and an aggregate quantity of labor N such that

1. *The policy function $g(k,s)$ solves the household's optimization problem with the return on capital given by $r = \tilde{r} - \delta$;*
2. *The stationary distribution $\lambda(a,s)$ is induced by $(\mathcal{P}, \bar{s})$ and $g(a,s)$, satisfying*
$$\lambda(k', s') = \sum_{j=1}^{m} \sum_{\{i: k' = g(k_i, \bar{s}_j)\}}^{n} Prob(s' | s = \bar{s}_j) \cdot \lambda(k_i, \bar{s}_j)$$
3. *The aggregate factors of production are given by*
$$K = \sum_{k,s} \lambda(k,s) g(k,s)$$
$$N = \xi'_{\infty} \bar{s}$$
where ξ_{∞} is the invariant distribution associated with $(\mathcal{P}, \bar{s})$;
4. *The prices satisfy*
$$w = F_N(K,N) \qquad and \qquad r = F_K(K,N) - \delta$$

Aiyagari's Equilibrium Computation

1. Set initial value of aggregate capital stock, K_0.
2. For the current iterate j, calculate the factor prices as
   $w_j = F_N(K_j, N), \quad r_j = F_K(K_j, N)$
3. Given the factor prices, solve the Bellman equation, which yields the decision rule $g_j(k, s)$.
4. Find the stationary distribution $\lambda_j(k, s)$.
   4.1 Start with an initial distribution.
   4.2 Move the mass of agents at each point in the state space to a new location in the state space, using $g_j(k, s)$ and the Markov process for s.
   4.3 Iterate to convergence.
5. Calculate
   $$K_j^* = \sum_{k,s} \lambda_j(k_i, \bar{s}_h) g_j(k_i, \bar{s}_h)$$
6. Set the next iteration of aggregate capital as
   $$K_{j+1} = \mu K_j^* + (1 - \mu) K_j$$
   where μ is the relative weight on K_j^* in the updating formula
7. Iterate until K_j^* is sufficiently close to K_j.

Additional Comments

- We can represent both models into a unified diagram.
  - Where is the buffer stock?
  - What are the effects of relaxing the constraint?
  - Capital is higher under incomplete markets.
    - What about welfare?
- How can we think about the lower bound of the grid?

Heterogeneous Agent's With Aggregate Risk

- There will no longer be a stationary distribution.
  - Stationary equilibrium can no longer be the solution concept.
- The distribution $\lambda_t(k, s)$ also becomes part of the state space.
  - Agents need to know that to forecast future factor prices.
  - *Curse of dimensionality*.
- Krusell and Smith (1998) propose a computational approach.