Notes IV - Real Business Cycle Models

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Macroeconomic Theory II (Ph.D.)

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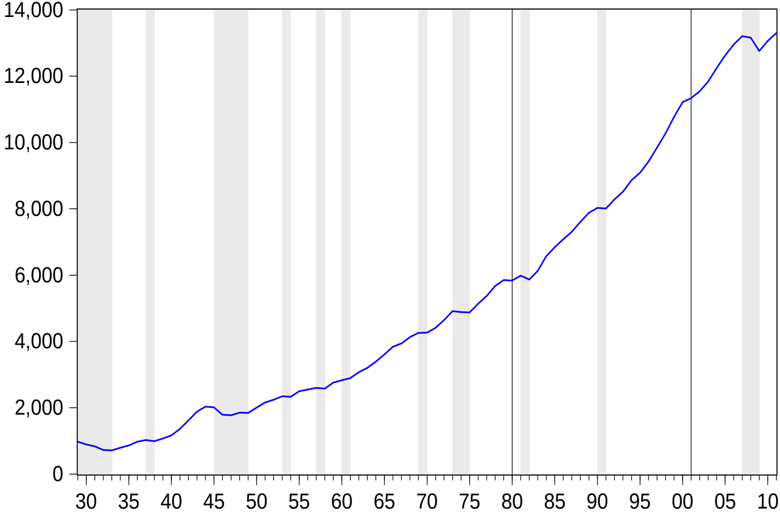
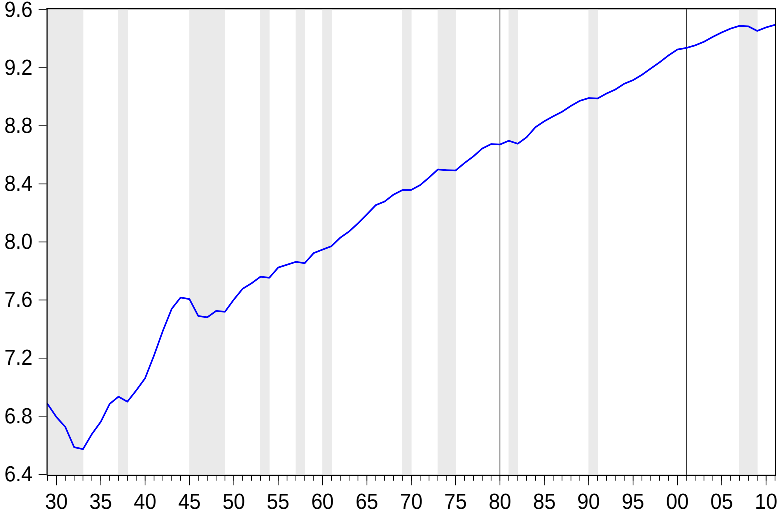
Outline

1. Log-Linearization
2. Basic RBC Model
3. Extensions:
    3.1 Indivisible Labor
    3.2 Variable Factor Utilization
    3.3 Money in the Utility Function
    3.4 Cash in Advance
    3.5 Imperfect Competition

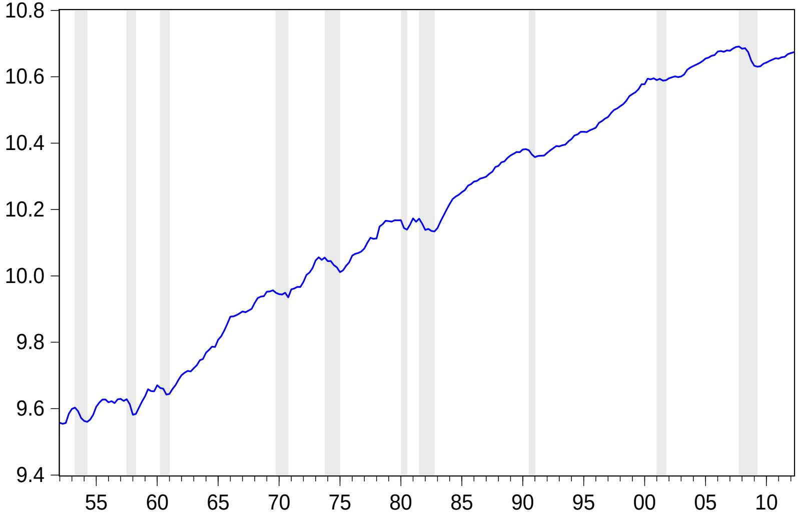
GDP Annual (Level)



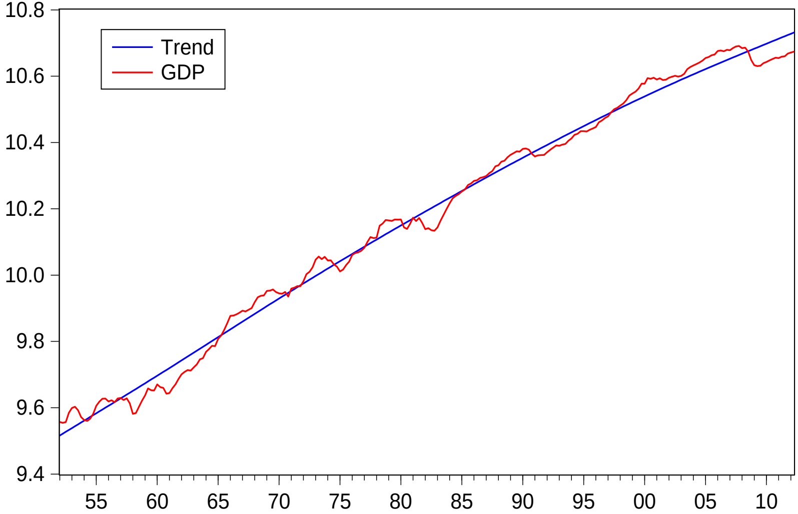
GDP Annual (Log)



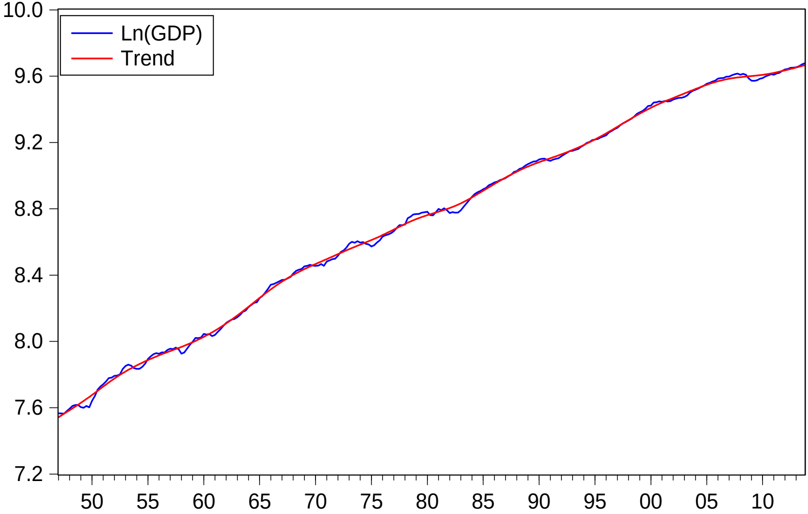
GDP Quarterly



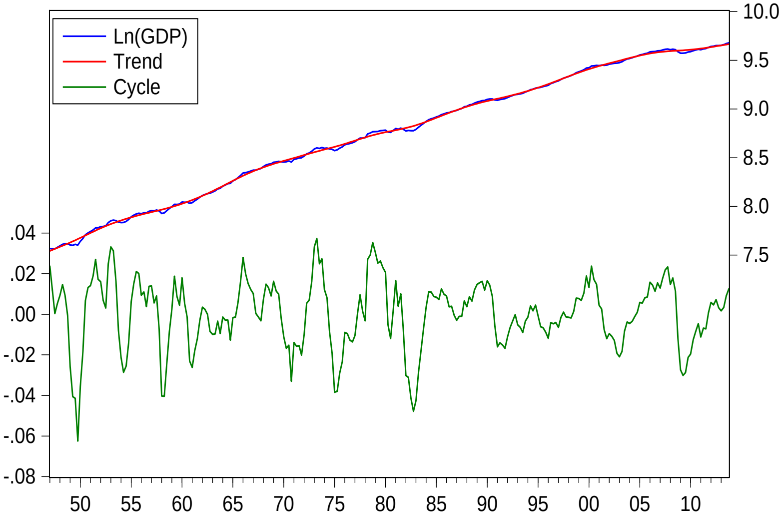
GDP & Log-Linear Trend



GDP & HP Trend

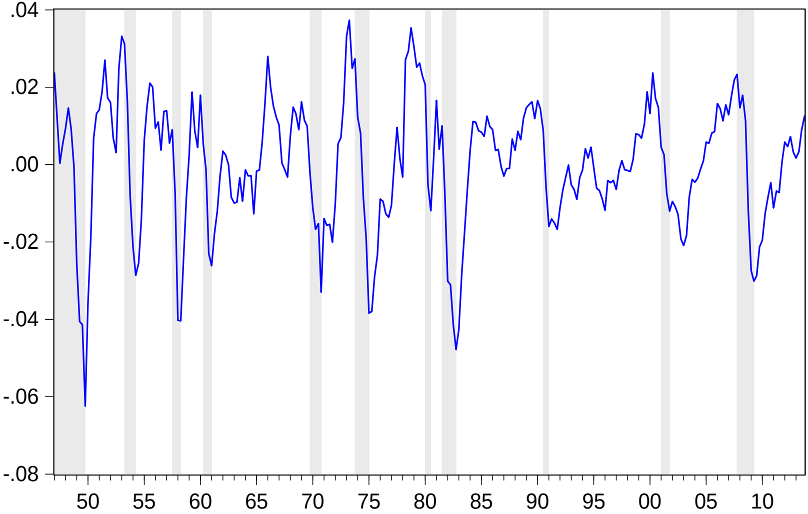


10.0
9.6
9.2
8.8
8.4
8.0
7.6
7.2
Ln(GDP)
Trend
50
55
60
65
70
75
80
85
90
95
00
05
10



Ln(GDP)
Trend
Cycle
10.0
9.5
9.0
8.5
8.0
7.5
.04
.02
.00
-.02
-.04
-.06
-.08
50
55
60
65
70
75
80
85
90
95
00
05
10

Cycle



That the predictions [of Keynesian economics] were wildly incorrect, and that the doctrine on which they were based was fundamentally flawed, are now simple matters of fact, involving no subtleties in economic theory. The task which faces contemporary students of the business cycle is that of sorting through the wreckage, determining what features of that remarkable intellectual event called Keynesian Revolution can be salvaged and put to good use, and which others must be discarded.
– *Lucas & Sargent (1978)*

Business Cycle Facts

Business Cycle Facts

U.S. Data: 1948Q1 - 2014Q3						
	σ	$\tilde{\sigma}$	ρ	$\rho_{x,y}$	$\rho_{x,y_{t-4}}$	$\rho_{x,y_{t+4}}$
y	0.017	1.00	0.85	1.00	0.07	0.11
c	0.009	0.53	0.79	0.76	0.07	0.22
I	0.047	2.76	0.87	0.79	-0.10	0.26
n	0.019	1.12	0.90	0.88	0.29	-0.03
y/n	0.011	0.65	0.72	0.42	-0.50	0.35
w	0.009	0.53	0.73	0.10	-0.10	0.10
$1+r$	0.004	0.24	0.42	0.00	0.27	-0.25
P	0.009	0.53	0.91	-0.13	0.09	-0.41
TFP	0.012	0.71	0.75	0.76	-0.34	0.34

- A word on TFP.

Quantitative Analysis

Calibration I

- β

$$\beta = \frac{1}{1+r}$$

- α

$$1-\alpha = \frac{wn}{y}$$

- δ

$$\delta = \frac{I}{k}$$

- θ

$$\theta = \frac{\frac{1-n}{n}(1-\alpha)\left(\frac{k}{n}\right)^{\alpha}}{\left(\frac{k}{n}\right)^{\alpha} - \delta\frac{k}{n}}$$

Calibration II

- We have

$$\ln \hat{z}_t = \ln y_t - \alpha \ln k_t - (1-\alpha) \ln n_t$$

- From the regression

$$\ln \hat{z}_t = \phi_0 + \phi_1 t_t + u_t$$

  - $\phi_0 = 0.147$ and $\phi_1 = 0.003$.
- Then I estimate an AR(1):

$$\hat{u}_t = \rho \hat{u}_{t-1} + \epsilon_t$$

  - $\rho = 0.974$ and $\sigma_\epsilon = 0.009$.

Data and Model

Data						
	σ	$\tilde{\sigma}$	ρ	$\rho_{x,y}$	$\rho_{x,y_{t-4}}$	$\rho_{x,y_{t+4}}$
y	0.017	1.00	0.85	1.00	0.07	0.11
c	0.009	0.53	0.79	0.76	0.07	0.22
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P	0.009	0.53	0.91	-0.13	0.09	-0.41
TFP	0.012	0.71	0.75	0.76	-0.34	0.34
Simulated Business Cycles						
	σ	$\tilde{\sigma}$	ρ	$\rho_{x,y}$	$\rho_{x,y_{t-4}}$	$\rho_{x,y_{t+4}}$
y	0.016	1 .00	0.73	1.00	0.13	0.13
c	0.007	0.44	0.78	0.95	0.34	-0.03
I	0.050	3.13	0.71	0.99	0.04	0.20
n	0.007	0.44	0.71	0.98	0.00	-0.23
y/n	0.010	0.63	0.23	0.99	-0.50	0.06
w	0.010	0.63	0.74	0.99	0.23	0.06
$1+r$	0.001	0.06	0.71	0.96	-0.05	0.26
TFP	0.012	0.75	0.72	0.99	0.11	0.15

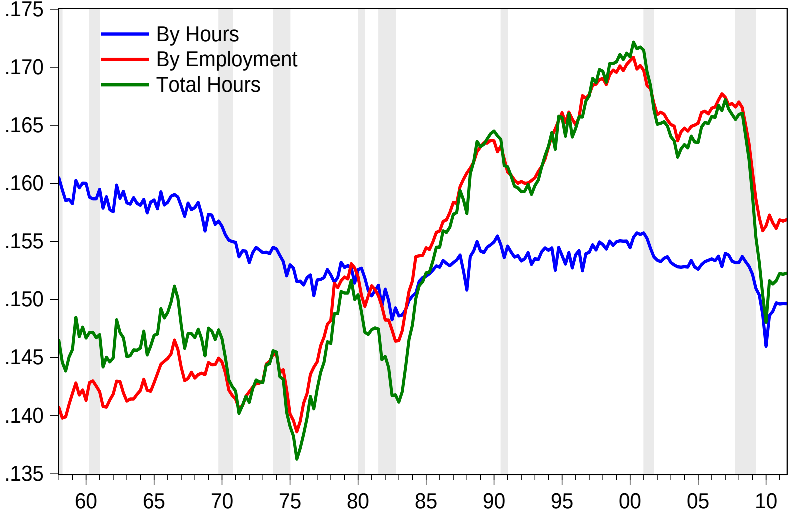
Evaluation

- The good: quantities.
- The bad: prices.
- The ugly: we need high frequency variation in z_t.
  - In absence of movements in k and z, labor and consumption must move in opposite directions.
- Overall, both the *amplification* and the *propagation* of the model are weak.

Extensions

Indivisible Labor

Where is the Volatilities in Hours Coming From?



Indivisible Labor

- Based on Hansen (1985) & Rogerson (1988).
- Households have two decisions:
  1. Whether to work or not.
  2. Conditional on working, how much to work.
- There is a large number of identical households.
- Each period households work $\bar{n}$ hours with probability τ_t.
  - Households can only choose probability of working.
- There is perfect insurance.
  - Every household gets paid whether they work or not.
- In expectation, households will work $n_t = \tau_t \bar{n}$
- The introduction of lotteries smooths out the goods consumption set.

Preferences

- Within period preferences are:

$$u(c_t, 1 - n_t) = \ln c_t + \theta \frac{(1 - n_t)^{1-\xi} - 1}{1 - \xi}$$

  - Frisch labor supply elasticity given by $(\xi\gamma)^{-1}$.

- Household expected flow utility is:

$$u(c_t, 1 - n_t) = \ln c_t + \tau_t \theta \frac{(1 - \bar{n})^{1-\xi} - 1}{1 - \xi} + (1 - \tau_t)\theta \frac{(1)^{1-\xi} - 1}{1 - \xi}$$

- With $\xi > 0$, we can write

$$u(c_t, 1 - n_t) = \ln c_t - B n_t$$

where $B = \frac{\theta}{\bar{n}} \left[\frac{(1)^{1-\xi} - 1}{1-\xi} - \frac{(1-\bar{n})^{1-\xi} - 1}{1-\xi} \right]$.

Households and Firms' Problem

- Households:

$$\max_{\{c_t, n_t, b_{t+1}, k_{t+1}\}_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\ln c_t - Bn_t)$$

subject to:

$$c_t + k_{t+1} - (1-\delta)k_t + b_{t+1} = w_t n_t + R_t k_t + (1+r_t)b_t$$

- Firms:

$$\max_{n_t, k_t} V_0 = z_t k_t^{\alpha} n_t^{1-\alpha} - w_t n_t - R_t k_t$$

First Order Conditions

$$1 = \beta \mathbb{E}_t \frac{c_t}{c_{t+1}}(1 + r_{t+1}) \quad (1)$$

$$B = \frac{1}{c_t} w_t \quad (2)$$

$$1 = \beta \mathbb{E}_t \frac{c_t}{c_{t+1}} \left[R_{t+1} + (1-\delta)\right] \quad (3)$$

$$w_t = (1-\alpha) z_t k_t^{\alpha} n_t^{-\alpha} \quad (4)$$

$$R_t = \alpha z_t k_t^{\alpha-1} n_t^{1-\alpha} \quad (5)$$

$$k_{t+1} = (1-\delta) k_t + I_t \quad (6)$$

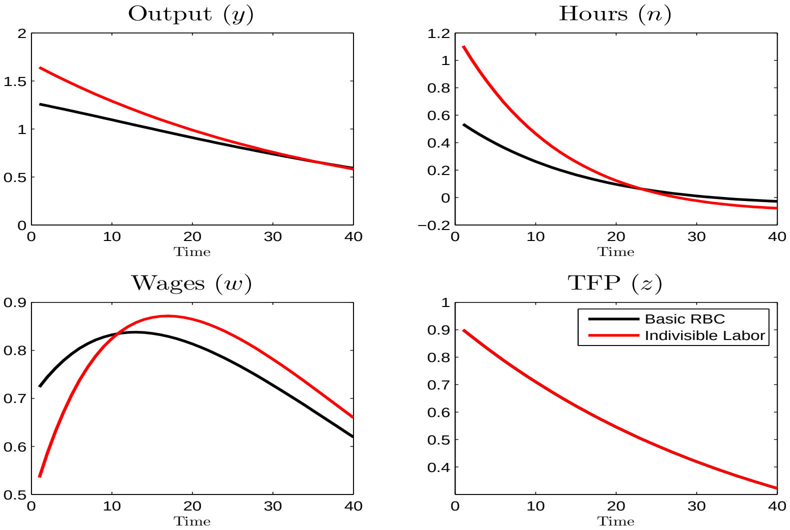
$$y_t = c_t + I_t \quad (7)$$

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha} \quad (8)$$

$$\ln z_t = \rho \ln z_{t-1} + \epsilon_t \quad (9)$$

- With our 'standard' calibration we can obtain $B = 2.63$.

Quantitative Results: IRFs



Quantitative Results: Second Moments

Moments			
	Data	Baseline RBC	Indivisible Labor
σ_y	0.017	0.0164	0.0214
$\tilde{\sigma}_n$	1.12	0.4268	0.6776
σ_w	0.53	0.0097	0.0078
$\rho_{w,y}$	0.10	0.9890	0.9251

Variable Factor Utilization

Variable Factor Utilization

- The basic RBC lacks amplification.
  - Only labor can make current output to go up.
- The idea is to allow capital to vary in response to shocks.
  - The intensity of use of current capital is also a choice.

Variable Factor Utilization

- Production is now given by

$$y_t = z_t(u_t k_t)^\alpha n_t^{1-\alpha}$$

- Also, if you work capital harder, it depreciates faster:

$$\delta_t = \delta_0 u_t^\phi \quad \phi > 1$$

Households and Firms' Problem

- Firm's own the capital.
- Household's problem is as before.
- Firm's problem:

$$\max_{n_t, k_{t+1}, u_t, I_t} V_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_t \left[z_t (u_t k_t)^{\alpha} n_t^{1-\alpha} - w_t n_t - I_t \right]$$

subject to:

$$k_{t+1} = \left(1 - \delta_0 u_t^{\phi}\right) k_t + I_t$$

First Order Conditions

$$1 = \beta\,\mathbb{E}_t \frac{c_t}{c_{t+1}}(1 + r_{t+1}) \tag{1}$$

$$\frac{\theta}{1 - n_t} = \frac{1}{c_t} w_t \tag{2}$$

$$1 = \beta\,\mathbb{E}_t \frac{c_t}{c_{t+1}} \left[\alpha z_{t+1} u_{t+1}^{\alpha} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} + (1 - \delta_0)\right] \tag{3}$$

$$w_t = (1 - \alpha) z_t (u_t k_t)^{\alpha} n_t^{-\alpha} \tag{4}$$

$$\alpha z_t u_t^{\alpha-1} k_t^{\alpha} n_t^{1-\alpha} = \delta_0 \phi u_t^{\phi-1} k_t \tag{5}$$

$$k_{t+1} = \left(1 - \delta_0 u_t^{\phi}\right) k_t + I_t \tag{6}$$

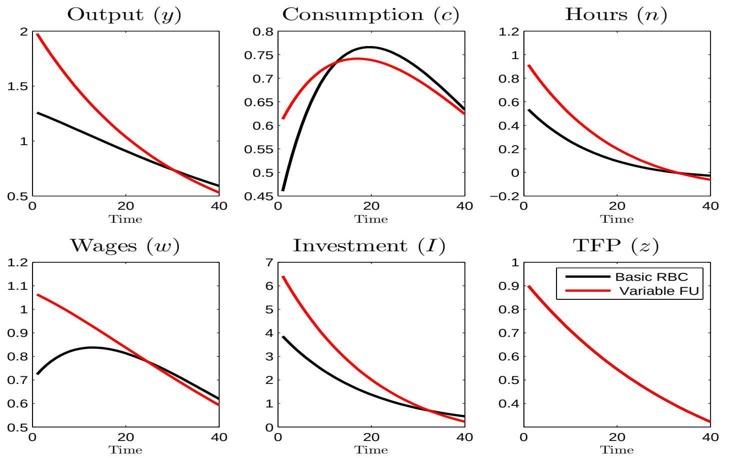
$$y_t = c_t + I_t \tag{7}$$

$$y_t = z_t (u_t k_t)^{\alpha} n_t^{1-\alpha} \tag{8}$$

$$\ln z_t = \rho \ln z_{t-1} + \epsilon_t \tag{9}$$

- Normalizing $u^* = 1$, we can use (5) to pin down ϕ.

Quantitative Results



Moments			
	Data	Baseline RBC	VFI
σ_y	0.017	0.0164	0.0258

Money

Money in an RBC Model

- Anything which:
  - is used as medium of exchange.
  - serves as unit of account.
  - serves as store of value.
- Not easy to get agents to hold money in equilibrium.
- Approaches:
  1. Money in the utility function.
     - Sidrauski (1967).
  2. Cash in advance.
     - Clower (1967), Svensson (1985), Lucas (1987), and Cooley & Hansen (1989).
  3. Money search.
     - Kiyotaki and Wright (1989).

Budget Constraint

- In terms of notation:
  - Nominal holding brought into period t.
  - Nominal price of goods.
  - Nominal interest rate.
  - Nominal bonds.
- Household income:
  1. Real income on work.
  2. Real income from leasing capital.
  3. Nominal interest earned on bonds.
- Household expenses:
  1. Consumption.
  2. Invest.
  3. Buy more (real) bonds.
  4. Accumulate more money.

Firms and Monetary Authority

- Firm problem is the standard static problem.
- There exists a central bank that sets the money supply.
  - With our timing, $\ln M_{t+1}$ is set at time t.
  - The exogenous process is given by

$$\ln M_{t+1} - \ln M_t = (1-\rho_m)\pi^* + \rho_m(\ln M_t - \ln M_{t-1}) + \epsilon_{m,t}$$

- This specification will generate positive trend inflation.
  - We will need to deal with this.

Money in the Utility Function

Money in the Utility Function

- Households also get utility from holding real money balances.

$$\max_{c_t, n_t, b_{t+1}, k_{t+1}, M_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln c_t + \theta \ln(1 - n_t) + \psi \frac{\left(\frac{M_{t+1}}{p_t}\right)^{1-\zeta} - 1}{1 - \zeta} \right]$$

subject to:

$$c_t + k_{t+1} - (1-\delta) k_t + \frac{B_{t+1}}{p_t} + \left(\frac{M_{t+1} - M_t}{p_t} \right) = w_t n_t + R_t k_t + (1 + i_t) \frac{B_t}{p_t}$$

- Firms:

$$\max_{n_t, k_t} V_0 = z_t k_t^{\alpha} n_t^{1-\alpha} - w_t n_t - R_t k_t$$

First Order Conditions

$$\frac{\theta}{1-n_t} = \frac{1}{c_t} w_t \tag{1}$$

$$1 = \beta \mathbb{E}_t \frac{c_t}{c_{t+1}} \left[(1+i_{t+1}) \frac{p_t}{p_{t+1}} \right] \tag{2}$$

$$1 = \beta \mathbb{E}_t \frac{c_t}{c_{t+1}} \left[R_{t+1} + (1-\delta) \right] \tag{3}$$

$$m_t = \psi^{\zeta} c_t^{\zeta} \left(\frac{1+i_{t+1}}{i_{t+1}} \right)^{\zeta} \tag{4}$$

$$w_t = (1-\alpha) z_t k_t^{\alpha} n_t^{-\alpha} \tag{5}$$

$$R_t = \alpha z_t k_t^{\alpha-1} n_t^{1-\alpha} \tag{6}$$

$$1 + r_{t+1} = (1+i_{t+1}) \mathbb{E}_t \frac{p_t}{p_{t+1}} \tag{7}$$

$$k_{t+1} = (1-\delta) k_t + I_t \tag{8}$$

$$y_t = c_t + I_t \tag{9}$$

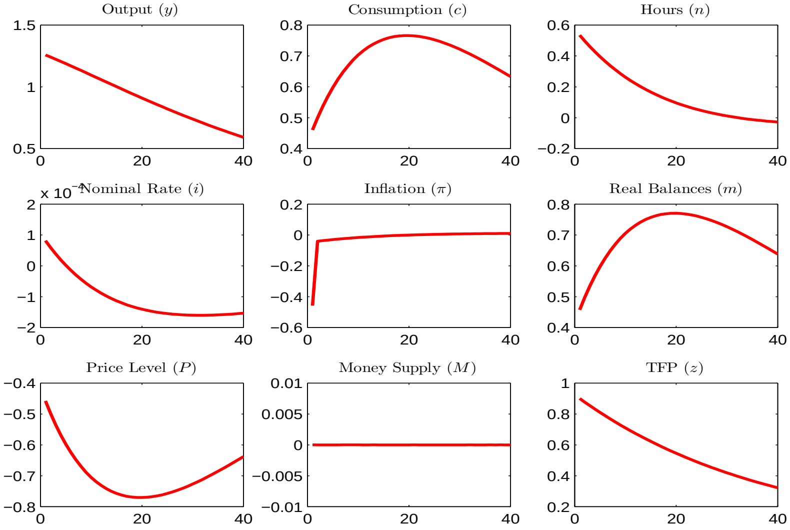
$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha} \tag{10}$$

$$\ln z_t = \rho \ln z_{t-1} + \epsilon_t \tag{11}$$

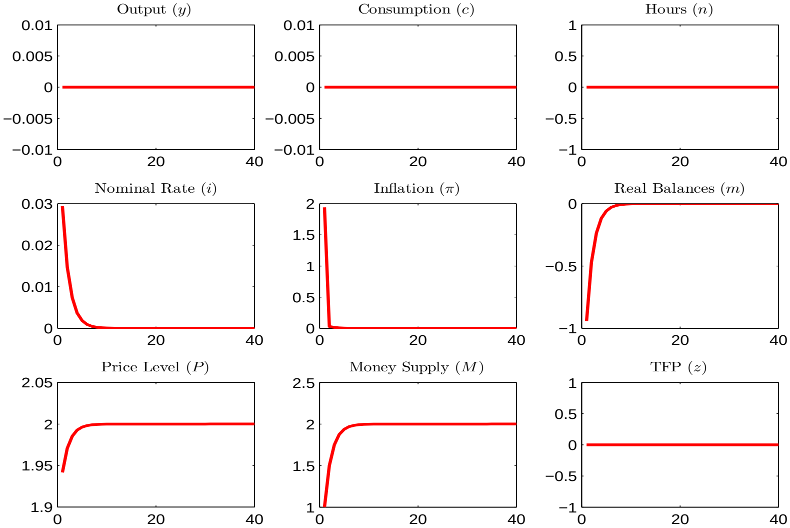
$$\Delta \ln m_t = \ln m_t - \ln m_{t-1} \tag{12}$$

$$\Delta \ln m_t = (1-\rho_m)\pi^* - \pi_t + \rho_m \pi_{t-1} + \rho_m \Delta m_{t-1} + \epsilon_{m,t} \tag{13}$$

Quantitative Results: TFP Shock



Quantitative Results: Monetary Policy Shock



Cash in Advance

Cash in Advance

- Assumption about transaction technology:
  - One must have enough money to finance nominal purchases of consumption goods:

$$M_t \geq p_t c_t$$

$$\max_{c_t, n_t, b_{t+1}, k_{t+1}, M_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln c_t + \theta \ln(1 - n_t)\right]$$

subject to:

$$c_t + k_{t+1} - (1-\delta)k_t + \frac{B_{t+1}}{p_t} + \frac{M_{t+1}}{p_t} = w_t n_t + R_t k_t + (1 + i_t)\frac{B_t}{p_t} + \frac{M_t}{p_t}$$

$$\frac{M_t}{p_t} \geq c_t$$

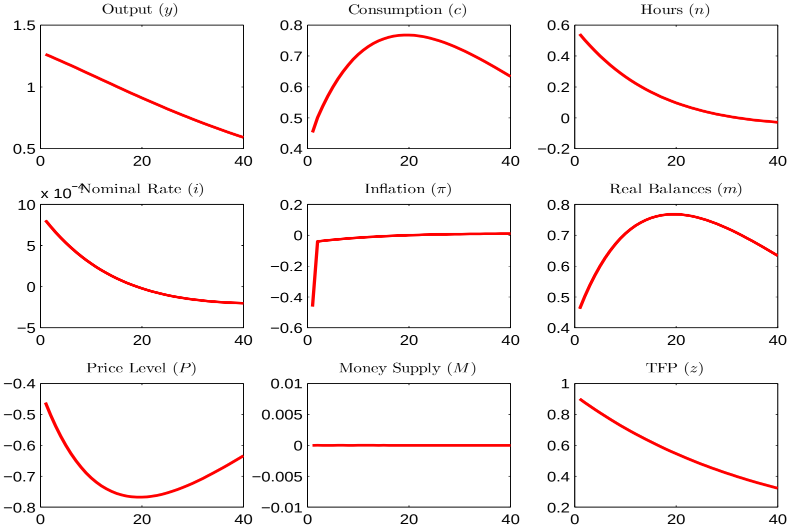
- Firms:

$$\max_{n_t, k_t} V_0 = z_t k_t^{\alpha} n_t^{1-\alpha} - w_t n_t - R_t k_t$$

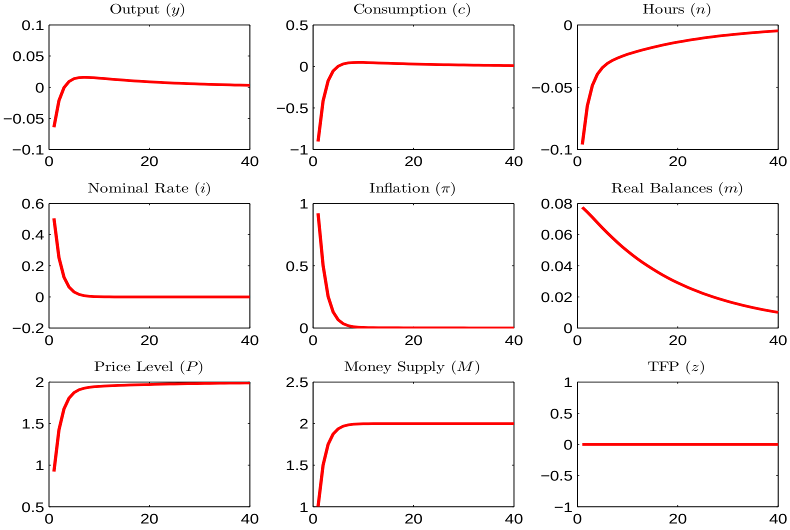
First Order Conditions

$$1/c_t = \lambda_t + \mu_t \tag{1}$$
$$\theta/(1-n_t) = (1/c_t)w_t \tag{2}$$
$$1 = \beta\,\mathbb{E}_t \frac{c_t}{c_{t+1}}\left[(1+i_{t+1})\frac{p_t}{p_{t+1}}\right] \tag{3}$$
$$1 = \beta\,\mathbb{E}_t \frac{c_t}{c_{t+1}}\left[R_{t+1} + (1-\delta)\right] \tag{4}$$
$$\lambda_t = \beta\,\mathbb{E}_t\left(\frac{\mu_{t+1}+\lambda_{t+1}}{1+\pi_{t+1}}\right) \tag{5}$$
$$w_t = (1-\alpha)z_t k_t^{\alpha} n_t^{-\alpha} \tag{6}$$
$$R_t = \alpha z_t k_t^{\alpha-1} n_t^{1-\alpha} \tag{7}$$
$$1 + r_{t+1} = \mathbb{E}_t \frac{1+i_{t+1}}{1+\pi_{t+1}} \tag{8}$$
$$m_{t-1} = (1+\pi_t)c_t \tag{9}$$
$$k_{t+1} = (1-\delta)k_t + I_t \tag{10}$$
$$y_t = c_t + I_t \tag{11}$$
$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha} \tag{12}$$
$$\ln z_t = \rho \ln z_{t-1} + \epsilon_t \tag{13}$$
$$\Delta \ln m_t = \ln m_t - \ln m_{t-1} \tag{14}$$
$$\Delta \ln m_t = (1-\rho_m)\pi^* - \pi_t + \rho_m \pi_{t-1} + \rho_m \Delta m_{t-1} + \epsilon_{m,t} \tag{15}$$

Quantitative Results: TFP Shock



Quantitative Results: Monetary Policy Shock



Discussion

- Monetary shocks have real (but tiny) effects in the CIA model.
  - Inflation is a tax.
    - Individuals can only substitute from money to leisure.
- Neither CIA nor MIU justify such a distortion.
  - What should the optimal policy be in this context?

Friedman Rule

- Inflation tax is directly related to the nominal interest rate.
  - Zero inflation tax is achieved with a zero nominal rate.
- The "Friedman rule" is just that: $i_{t+1} = 0$.
  - Money reduces transaction frictions and is costless to produce.
  - $i_{t+1} > 0$ imposes a tax on holders of money.
- The Friedman rule is optimal in both environments.

Imperfect Competition

First Order Conditions

$$1 = \beta\,\mathbb{E}_t\,\frac{c_t}{c_{t+1}}(1 + r_{t+1}) \tag{1}$$

$$\frac{\theta}{1 - n_t} = \frac{1}{c_t} w_t \tag{2}$$

$$1 = \beta\,\mathbb{E}_t\,\frac{c_t}{c_{t+1}}\left[R_{t+1} + (1 - \delta)\right] \tag{3}$$

$$w_t = \frac{\nu - 1}{\nu}(1 - \alpha) z_t k_t^{\alpha} n_t^{-\alpha} \tag{4}$$

$$R_t = \frac{\nu - 1}{\nu}\alpha z_t k_t^{\alpha - 1} n_t^{1-\alpha} \tag{5}$$

$$k_{t+1} = (1 - \delta) k_t + I_t \tag{6}$$

$$y_t = c_t + I_t \tag{7}$$

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha} \tag{8}$$

$$\ln z_t = \rho \ln z_{t-1} + \epsilon_t \tag{9}$$

Basic RBC vs. Imperfect Competition

- Imperfect competition is a steady state distortion.
  - In a linearized world, IRFs would be identical.
    - To a first order approximation, the dynamics are the same.
- Taxes can restore the first best equilibrium.
- What about fluctuations in the markup?
  - Let's suppose:

$$\ln \varphi_t = (1 - \rho_\varphi)\varphi^* + \rho_\varphi \ln \varphi_{t-1} + \epsilon_{\varphi,t}$$

First Order Conditions With Markup Shocks

$$1 = \beta\,\mathbb{E}_t\,\frac{c_t}{c_{t+1}}(1 + r_{t+1}) \tag{10}$$

$$\frac{\theta}{1 - n_t} = \frac{1}{c_t} w_t \tag{11}$$

$$1 = \beta\,\mathbb{E}_t\,\frac{c_t}{c_{t+1}}\left[R_{t+1} + (1 - \delta)\right] \tag{12}$$

$$w_t = \frac{1}{\varphi_t}(1 - \alpha) z_t k_t^{\alpha} n_t^{-\alpha} \tag{13}$$

$$R_t = \frac{1}{\varphi_t}\alpha z_t k_t^{\alpha - 1} n_t^{1-\alpha} \tag{14}$$

$$k_{t+1} = (1 - \delta) k_t + I_t \tag{15}$$

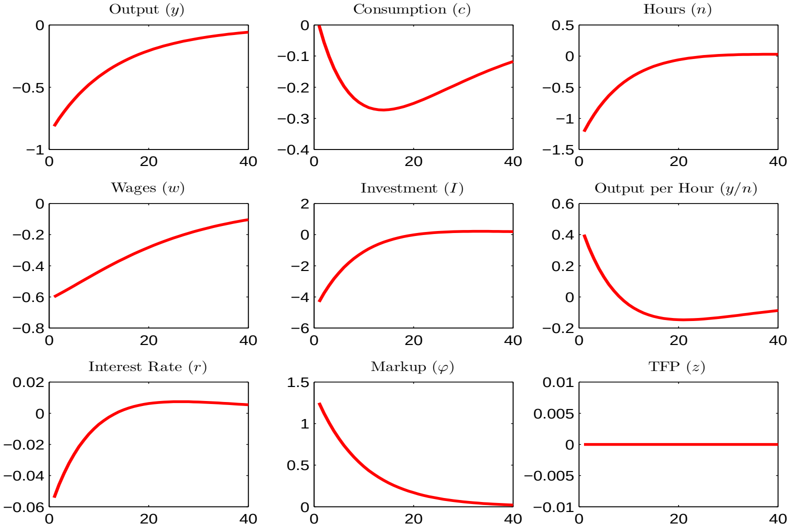
$$y_t = c_t + I_t \tag{16}$$

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha} \tag{17}$$

$$\ln z_t = \rho \ln z_{t-1} + \epsilon_t \tag{18}$$

$$\ln \varphi_t = (1 - \rho_\varphi)\varphi^* + \rho_\varphi \ln \varphi_{t-1} + \epsilon_{\varphi,t} \tag{19}$$

Quantitative Results: Markup Shock



Taking the Calibration More Seriously

- From

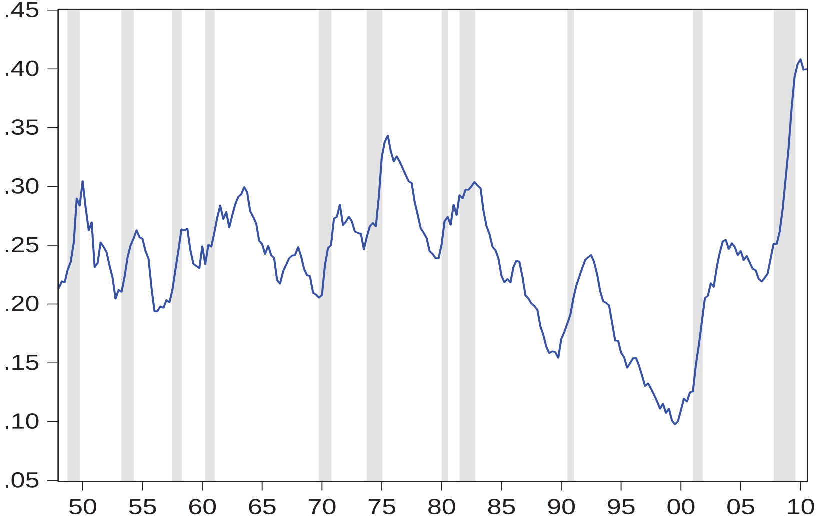
$$\frac{\theta}{1-n_t} = \frac{1}{c_t}\frac{1}{\varphi_t}(1-\alpha)\frac{y_t}{n_t}$$

- I can estimate

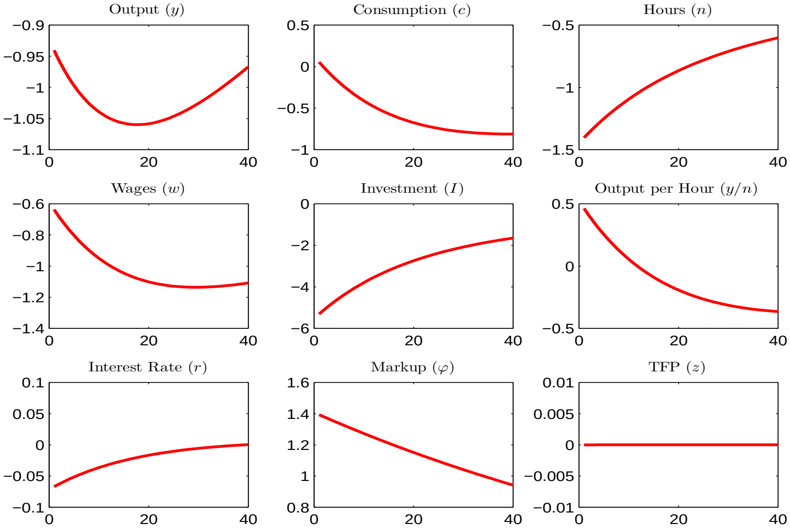
$$\ln \hat{\varphi}_t = \beta_0 + \rho_\varphi \ln \hat{\varphi}_{t-1} + \epsilon_{\varphi_t}$$

  - $\varphi^* = 1.26$.
  - $rho_\varphi = 0.99$.
  - $\sigma_{\epsilon_{\varphi_t}} = 0.011$.

Markup Shock



Quantitative Results: Estimated Markup Shock



Business Cycle Moments

	Data		RBC		RBC w/Markup Shocks	
	$\tilde{\sigma}$	$\rho_{x,y}$	$\tilde{\sigma}$	$\rho_{x,y}$	$\tilde{\sigma}$	$\rho_{x,y}$
y	1	1	1	1	1	1
c	0.53	0.76	0.44	0.95	0.4	0.68
n	1.12	0.88	0.43	0.98	0.95	0.82
I	2.76	0.79	3.07	0.99	4.38	0.95
y/n	0.65	0.42	0.63	0.99	0.6	0.37
TFP	0.71	0.76	0.72	0.99	0.59	0.79

- Do we think that fluctuations in *nu* are important?
- Chari, Kehoe, McGrattan (2008): Business Cycle Accounting.
  - Wedges as a guidance.
  - If not markups, then what?