Macroeconomic Theory II
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Problem Set 5

Due Date: Friday, April 26.
Instructions: Use LaTeX.

1. The individual's problem is described by:

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta u(c_t)$$

subject to:

$$c_t + a_{t+1} = (1+r)a_t + y_t$$
$$a_{t+1} \geq -\bar{a}$$

   (a) Set up the Bellman equation.

   (b) Assume utility is quadratic and of the form $u(c) = bc_t - \frac{1}{2}dc_t^2$. Show that in the presence of borrowing constraints, income uncertainty declines present consumption.

   (c) Define 'prudence', in the context of preferences. What would have happened if utility was such that prudence behavior was present?

2. **A Simple Buffer-Stock Model.** Consider an economy with consumers whose objective is to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

with $\sigma = 2.0$ and $\beta = 0.9$. There is only one asset, which yields a constant, risk-free rate of return, $r = 0.07$. Each consumer receives an idiosyncratic, stochastic endowment of income, y_t, which follows an AR(1) process with a mean of 10, a first-autocorrelation $\rho = 0.85$, and a standard deviation of 1 for the error term.

   (a) Write down the Bellman equation that describes the consumers' optimization problem.

   (b) What is the 'natural debt limit' for this problem?

   (c) Use the program `markovappr.m` to approximate the AR(1) income process using a 9-state Markov process. You will need the transition matrix and the vector of values below.

   (d) Solve for the value function and optimal policy function by iterating on the Bellman equation. Do this by discretizing the range of values for asset holdings (initially use just 100 grid points and then once your code is working, increase it to 500 grid points). Make sure that you are choosing appropriate values for the lower and upper values of the grid (and continue to be sure of this as you alter the model in questions below!).

   (e) Plot both the consumption function and next period's asset holdings as a function of this period's asset holdings.

   (f) Simulate 1,000 samples with 1,000 periods of income and consumption "data" and calculate the means, standard deviations, and first-autocorrelations, and their cross-correlation for each sample. Then calculate the means and standard errors of those statistics (using the 1,000 samples). Report them in an easy-to-read table.

   (g) Plot 150 periods of income, asset holdings, and consumption from one sample.

(h) Now suppose that you have a very large number of (*ex ante* identical) agents who are solving this same problem. Because they each receive different income shocks each period, they will look different *ex post*—some will have good luck and accumulate significant assets, others will have poor luck and draw down their assets. There will be a stationary distribution of agents' wealth holdings—an individual's location in the distribution is changing, but the overall distribution is stationary. Calculate and plot the stationary distribution of wealth. Compute mean and standard deviation of this distribution of wealth holdings.

(i) Now impose a borrowing constraint of $a_t \geq 0, \forall t$ (and continue to assume this for the remainder of the problem set), and re-solve the model (repeat steps 2d–2h). Demonstrate that this borrowing constraint, in contrast with the "natural debt limit," actually binds. Briefly discuss/explain the results.

(j) Allow changes in income to be more persistent, i.e. change its first-autocorrelation to 0.95. Re-do parts 2d–2h. Briefly discuss/explain the results.

(k) Assuming once again the original transition matrix (with $\rho = 0.85$), now increase risk aversion (set $\sigma = 3$) and re-do parts 2d–2h again. Briefly discuss/explain the results (via comparison with the original results).

(l) Assuming once again $\sigma = 2$, now increase impatience (set $\beta = 0.88$) and re-do parts 2d–2h a final time. Briefly discuss/explain the results (via comparison with the original results).

3. **Borrowing Limits.** A consumer has preferences over sequences of a single consumption good that are ordered by $\sum_{t=0}^{\infty} \beta^t u(c_t)$, where $\beta \in (0,1)$ and $u(\cdot)$ is strictly increasing, twice continuously differentiable, strictly concave, and satisfies the Inada condition $\lim_{c \downarrow 0} u'(c) = +\infty$. The consumer has an endowment sequence of the one good: $y_t = \lambda^t, \quad \forall t \geq 0$, where $|\lambda\beta| < 1$. The consumer can lend at a constant and exogenous risk-free interest rate of r that satisfies $(1+r)\beta = 1$. The consumer's budget constraint at time t is

$$\frac{1}{1+r} a_{t+1} + c_t \leq y_t + a_t$$

for all $t \geq 1$, where a_t is the consumer's asset holdings at the beginning of period t. Assume that initial assets are $a_0 = 0$ and that $y_0 = 1$. Assume that the consumer is subject to the *ad hoc* borrowing constraint $a_t \geq 0, \quad \forall t \geq 1$.

(a) Write down the consumption Euler equation for this model.

(b) When thinking about consumption growth, does it matter in this model whether $u'''(c) > 0$? Explain.

(c) Assume that $\lambda > 1$ and compute the consumer's optimal plan for $\{c_t, a_{t+1}\}_{t=0}^{\infty}$. What is the growth rate of consumption? What is the growth rate of assets?

(d) Assume now that $\lambda < 1$ and compute the consumer's optimal plan for $\{c_t, a_{t+1}\}_{t=0}^{\infty}$.

(e) Compute the natural borrowing limits $\forall t \geq 0$.