Macroeconomic Theory II
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Problem Set 4

Due Date: Beginning of class on Thursday, March 16.
Instructions: Use LaTeX.

1. **Stochastic Growth Model.** Consider the standard stochastic growth model with technology given by $y_t = z_t k_t^{\alpha}$ and agents that maximize lifetime expected utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

where ρ represents the subjective discount rate. Baseline parameter values are $\rho = 0.05$, $\alpha = 0.33$, $\delta = 0.01$, and $\sigma = 2$. The stochastic process for z_t is a three-state markov process with $\bar{z} = \begin{bmatrix} 0.9 & 1.0 & 1.1 \end{bmatrix}$ and transition matrix

$$P = \begin{bmatrix} .7 & .3 & 0 \\ .3 & .4 & .3 \\ 0 & .3 & .7 \end{bmatrix}.$$

   (a) Set this problem up as a dynamic programming problem. What is/are the state variable/s? What is/are the control variable/s? Write down the Bellman equation.

   (b) Write a Matlab code that solves the Bellman equation by value function iteration.[1] Then, carry out the following analysis:

      i. Simulate 5,000 periods of 'data' for capital, output, consumption, and investment, using an initial capital stock of $k_0 = k^*$. This will require that you use interpolation to deal with points between your gridpoints for k. Using 500 periods as *burn-in*, plot two hundred periods of the four series (perhaps use `subplot` to stack four separate plots vertically).

      ii. Use these data to calculate the means, standard deviations, cross correlations, and first autocorrelations for consumption, the capital stock, output, and investment.

      iii. Discuss and interpret the economics behind these statistics.

   (c) Plot a histogram of the values of k for simulations 501 to 5,000. This gives you an approximation of the ergodic distribution of capital per worker in the long run.

   (d) Repeat the steps in 1b, but with $\sigma = 5$. Comment on how the statistics have changed relative to the previous part, providing economic intuition.

   (e) Set $\sigma = 2$ again. Now change the code so that capital investments are irreversible.Repeat the steps in 1b again, being sure to discuss and interpret the economics behind the results—specifically how they changed relative to the first part.

2. **Aggregate Series and the Hodrick-Prescott Filter.** From the BEA download quarterly, seasonally adjusted data on US real GDP from 1950Q1 to 2015Q4. Take natural logs of the data, and then with the code `hp_filter.m` obtain the cyclical component of the transformed series using the Hodrick-Prescott filter with a smoothing parameter $\lambda = 1600$, which is the standard value used for quarterly series.

[1] For the grid of the capital stock, use a minimum value 0.25 times the steady state capital stock and a maximum value 1.75 times the steady state capital stock, with 300 grid points between.

(a) Show a plot of the cyclical component and a plot of the trend component.

(b) What is the standard deviation of the HP-detrended GDP?

(c) Now, use the HP Filter to detrend the simulated output obtained from the stochastic growth model in Part 1 and calculate the standard deviation of the cyclical component. How does the this value compares with the one calculated from the 'raw' data? Why didn't we detrended the data simulated from the stochastic growth model in Question 1?