Macroeconomic Theory II
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Spring 2017
Problem Set 1

Due Date: Beginning of class on Thursday, January 28.
Instructions: Use LaTeX. Label all plots, series, and axes accordingly.

1. **First-order linear difference equation**. Consider the difference equation $x_t = \frac{1}{5}x_{t-1} + 8$.
   (a) Find the steady-state of this difference equation.
   (b) Choose a starting value $x_0 = 15$, which is above the steady state, and then iterate on the difference equation 50 times. Plot the resulting path for x_t in a graph with the time period on the horizontal axis. Add to the plot a horizontal line at the steady-state value, so that you can see x_t converging to the steady-state.
   (c) Repeat the previous step, but use a starting value of $x_0 = 5$, which is below the steady state.
   (d) Solve the difference equation by hand, using $x_0 = 5$ as the starting value. Using the solution that you derived by hand, compute the first 50 values in Matlab and confirm that the solution gives the same values as the iteration procedure in part 1c.
   (e) Now consider the difference equation $x_t = 0.9x_{t-1} + 1$. Note that the steady state is the same as for the previous equation, but that the coefficient on x_{t-1} is closer to 1. Use a starting value of $x_0 = 15$ and iterate for 50 periods. Plot the result and note the speed of convergence relative to the initial difference equation.

2. **System of two first-order linear difference equations.** Consider the following system of difference equations:

$$x_{1,t} = 0.75x_{1,t-1} + 0.15x_{2,t-1} + 3$$
$$x_{2,t} = 0.65x_{1,t-1} + 0.15x_{2,t-1} + 2.$$

   (a) Compute the steady states for x_1 and x_2.
   (b) Use the `eig()` command to get the eigenvalues and eigenvectors of this system.
   (c) Use the eigenvalues and eigenvectors to transform the system into a diagonal system.
   (d) Analytically determine the solution to the two first-order difference equations of the transformed diagonal system.
   (e) Using this transformed system, express the analytic solution in terms of the original variables.
   (f) Using $x_{1,0} = 10$ and $x_{2,0} = 10$, compute 50 periods of values for $x_{1,t}$ and $x_{2,t}$ using the analytic solution.
   (g) Now compute 50 periods of values for $x_{1,t}$ and $x_{2,t}$ by iterating on the system above.
   (h) Show that the iterative approach and the analytic solution yield identical results.
   (i) Plot the paths for $x_{1,t}$ and $x_{2,t}$, along with lines indicating the steady states of the two variables.

3. **Stochastic first-order linear difference equation.** Consider the AR(1) process:

$$y_t = 1 + 0.9y_{t-1} + \epsilon_t.$$

   (a) What is the unconditional mean?

(b) Assume a starting value that is significantly above that mean: y_0=20. Simulate 100 periods of data by drawing random shocks for ϵ_t from a normal distribution with mean zero and standard deviation 0.5 (use the Matlab command `randn` to generate the random draws).

(c) Plot the 100 observations. From viewing the plot, after approximately how many periods does the process appear to be stationary?

(d) Now consider the following process

$$y_t = 5 + 0.5y_{t-1} + \epsilon_t$$

Note that the unconditional mean for this AR(1) is the same as above. Again simulate 100 periods and plot the data.

(e) Does the data now start to look stationary sooner? How does this relate to the assumed persistence of the AR(1) process?

4. **Markov processes.** x_t is a 3-state Markov chain that takes on values in $[3\ \ 4\ \ 5]$ and has a transition matrix

$$P = \begin{bmatrix} .7 & .3 & 0 \\ .3 & .4 & .3 \\ 0 & .3 & .7 \end{bmatrix}.$$

(a) Compute the stationary distribution associated with this transition matrix.

(b) Show a very quick way (needing just one matrix operation) to check your answer in 4a.

(c) Now use an iterative procedure to calculate the stationary distribution.

(d) Use the Matlab function `markovvec.m` to simulate 10,000 observations of the Markov chain. Use these observations to confirm the stationary distribution in yet another way.

(e) If $x_t = 3$, what is the probability that $x_{t+5} = 5$?

(f) Assuming that the initial probability distribution (π_0) is equal to the stationary distribution, what is the unconditional expectation of x_t?

5. **Simulating an exogenous endowment stream.**[1] The growth rate of GDP, x_t, is, approximately, given by $x_t = \ln y_t - \ln y_{t-1}$. Assume that the random variable $\{X_t\}$ follows a 3-state Markov chain with

$$\begin{aligned} x &= (\mu - \sigma, \mu, \mu + \sigma) \\ &= (-0.02, 0.02, 0.04). \end{aligned}$$

(note that x_t represents the time-t realization of the random variable X_t).[2] The transition matrix of the Markov process is given by:

$$P = \begin{bmatrix} .5 & .5 & 0 \\ .03 & .9 & .07 \\ 0 & .2 & .8 \end{bmatrix}.$$

(a) Interpret the elements in the transition matrix (e.g. from the point of view of this economy, what does it mean that conditional on being in state 2 there is a 90% probability of staying there?)

(b) How can the level of output in this economy be recovered from the growth rates? Hint: it involves solving a rearrangement of the difference equation given above.

[1] Thanks to Chris Edmond for sharing this.

[2] The idea here is that the economy can either be shrinking ($x = \mu - \sigma < 0$), growing at a usual pace ($x = \sigma > 0$), or growing even faster ($x = \mu + \sigma$).

(c) Assume that $\pi_0' = [0\ 1\ 0]$ and use the Matlab function `markovvec.m` to simulate 200 periods of growth rates for this economy. Assume $\ln y_0 = 0$ as the initial condition.

(d) Drop the first 100 periods. Why would you like to do that?

(e) Plot the realized growth rates of GDP as well as its implied level.