Notes V - Equilibrium

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Intermediate Macroeconomics
Fall 2017

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General Equilibrium

- Level of analysis:
  1. Decision theory.
     - How agents make decisions, given prices.
  2. Partial equilibrium.
     - How does a price clears one market, taking all other prices as given.
  3. General equilibrium.
     - How do all prices work to clear all markets simultaneously.
- Several endogenous variables have been treated as exogenous.
  - Let's start changing that.
- What has been the only price we have encountered thus far?
- We are going to introduce an equilibrium concept and endogenize the real interest rate.
  - Later, we are going to endogenize production.

Endogenizing the Interest Rate

Environment

- Economy populated by many identical agents.
- Normalize to one:
  - The representative agent.
- Agent lives for two periods and solves standard consumption-saving problem.
  - Takes income as given.
    - Endowment economy.
- For now, we are not going to pay attention to the "supply-side" of the economy.

Equilibrium

Definition (Competitive Equilibrium)

Set of prices and allocations such that:

1. *All agents in the economy are behaving optimally, taking prices as given.*
2. *Prices are such that markets clear.*

- What are the key ingredients here?
  - Prices adjust so quantity demanded equals quantity supplied when agents behave according the optimal decision rules.
- We focused on 1., now were are going to focus more on 2.
  - Supply still exogenous for now.

Demand Side

- We are moving away from individual's decision.
  - More towards understanding aggregates.
    - Denoted by capital letters.
  - Optimization problem:
    - Before.
    - Now?
- From the household problem we obtained the consumption function.
- Demand side: total desired expenditures equals total consumption.

$$Y^d = C(Y, Y', r)$$

  - Y is in both sides!
- Supply side: $Y^s = Y$.
  - Exogenously supply aggregate endowment.

Graphical Derivation of the Y^d Curve

- We can deal with the complication mentioned before.
  - A graph with a 45 degree line.
  - Y^d against Y.
  - What is the slope?
    - MPC.
    - Less than 1.

The "aggregate demand curve", or what we will also sometimes simply call the "Y^d curve" plots out how the quantity of goods demanded in aggregate varies with the real interest rate.

- How do changes in interest rates affect Y^d?
- Current consumption is decreasing in real interest rates.
  - Substitution effect dominates.
- How does that look in the (Y^d, r) space?
- Remember, supply of current goods, Y^s, doesn't change with r.
  - There is no production, so it is exogenous.

Comparative Statics

- Increase in current income: “supply shock”.
  - $\downarrow r$.
- Increase in future income: “demand shock”.
  - $\uparrow r$.
- Intuition:
  - Market-clearing condition.
  - Real interest rates has to “undo” desired changes in consumption.
  - Conditional on the real interest rate, people want to smooth.
    - Interest rate has to move to “prevent” smoothing.

Algebraic Example

- Assuming logarithmic utility we have household consumption:

$$c = \frac{1}{1+\beta}\left(y + \frac{y'}{1+r}\right)$$

- What about for aggregates?
- In equilibrium

$$Y = Y^s = Y^d = C$$

Therefore,

$$1 + r = \frac{1}{\beta}\frac{Y'}{Y}$$

  - This is just an Euler equation.
- The economy as a *whole* doesn't save.
- Interest rate is then a measure of how plentiful future is relative to present.
  - What if $Y = Y'$?

Adding Government

- Aggregate government spending is G and G'.
- Can tax, T and T'.
- Can borrow, S_G.
- What would be the government intertemporal budget constraint?
- What would be the household intertemporal budget constraint?
  - What if we combined both?
  - Taxes drop out!
    - Ricardian equivalence.
    - Makes no difference whether current spending financed with taxes or debt.

Consumption Function and Aggregate Demand

- Timing of taxes does not matter, but spending does. Household consumption function is:

$$C = C(Y - G, Y' - G', r)$$

- With logarithmic utility:

$$C(Y - G, Y' - G', r) = \frac{1}{1+\beta}\left(Y - G' + \frac{Y' - G'}{1+r}\right)$$

- To obtain the total demand for current goods:
  - Note that in equilibrium government borrowing must be equal to household saving: $S_G = -S$.
  - Combine the definition of government savings/borrowing with the household budget constraint.
- Aggregate demand relationship is given by

$$Y^d = C(Y - G, Y' - G', r) + G$$

Effects of Changes in Government Spending

- We have that

$$Y^d = C(Y - G, Y' - G', r) + G$$

- Increase in current government expenditure, $\uparrow G$.
  - $Y^d \nearrow$, $r \uparrow$, $c \downarrow$.
  - What are households doing with the remaining income?
  - Consider two scenarios:
    1. Increase in government expenditures financed entirely by taxes.
    2. Increase in government expenditures financed entirely by borrowing.
- Increase in future government expenditure, $\uparrow G'$.
  - $Y^d \swarrow$, $r \downarrow$, c unchanged.
- Intuition:
  - Remember what r was measuring.

Equilibrium in a Production Economy

Motivation

- So far: study equilibrium in an endowment economy.
- Now: we will study equilibrium in an economy with production.
  - Why?
- We will construct a model that can be used to compared to the actual behavior of the economy in the short run.
  - Why?
    - We can study the effects of different policies.

Equilibrium and Framework

- Definition of equilibrium is still the same:
  - Set of prices and quantities consistent with:
    1. Agents optimizing, taking prices as given.
    2. Markets clearing.
- Agents:
  1. Household.
  2. Firm.
  3. Government.
- Large number of each kind of agent.
  - Since they are identical:
    - Price taking behavior.
    - We can use the representative agent problem.
- Only two periods:
  - Present and future.

Firm

- Uses an increasing and concave c.r.s. production function that requires capital and labor:

$$Y = AF(K, N)$$

- Take real wage, w as given.
- Firms own the capital stock and make investment decisions.
  - Different from the Solow-Swan model.
  - This doesn't change the results.
- Capital goods differ from consumption goods.
  - Resources can be transferred across time.
  - When households save, saving is lent to firms, that then buy capital goods.
    - More items can be produced in the future.
- We will assume that capital can be converted back into consumption.
  - Only a two-period model and we don't want capital to be wasted.

Capital Accumulation, Firm Profit, and Firm Value

- Capital accumulation the same as before.

$$K' = qI + (1-\delta)K$$

- Terminal condition: $K'' = 0 \Rightarrow I' = -(1-\delta)K'$.
- Profits (or Dividends):

$$\Pi = Y - wN - I$$

- Firm value: present value of lifetime profits/dividends:

$$V = \Pi + \frac{1}{1+r}\Pi'$$

  - Saving at interest rate r is another way of transferring resources into the future.
- Firm chooses investment, capital, and labor to maximize V.

Firm's Problem

$$\max_{N,N',I,K'} V = zF(K,N) - wN - I + \frac{1}{1+r}\left[z'F(K',N') - w'N' + \frac{(1-\delta)}{q'}K'\right]$$

Subject to

$$K' = I + (1-\delta)K$$

Which can also be expressed as,

$$\max_{N,N',K'} zF(K,N) - wN - \frac{K'}{q} + \frac{(1-\delta)}{q}K + \frac{1}{1+r}\left[z'F(K',N') - w'N' + \frac{(1-\delta)}{q'}K'\right]$$

- FOCs.
  - N: $\partial V/\partial N = 0$
  - N': $\partial V/\partial N' = 0$
  - K': $\partial V/\partial K' = 0$

First Order Conditions

- Optimality Conditions.

$$\frac{\partial V}{\partial N} = 0 \Leftrightarrow zF_N(K, N) = w$$
$$\frac{\partial V}{\partial N'} = 0 \Leftrightarrow z'F_{N'}(K', N') = w'$$
$$\frac{\partial V}{\partial K'} = 0 \Leftrightarrow 1 = \frac{1}{1+r}\left[z'F_{K'}(K', N') + (1-\delta)\frac{q}{q'}\right]$$

- Intuition?
  - Marginal benefit = Marginal cost.
- Note that firm could:
  - Save and earn the market rate.
  - Invest and earn the MPK minus depreciation.
    - In the Solow model, the firm rental rate covered both interest and depreciation costs.

Labor Demand

- The first two first order conditions imply labor demand curves.

$$N^d(w, A, K)$$

- Labor demand (those equations) are "static".
  - Depend only on current period variables.
- Labor demand decreasing function of real wage.
- Labor demand shifts out if $\uparrow A$.
  - Technological improvement.
- Labor demand would shift if in if $\downarrow K$.
  - Natural disaster.

Investment Demand

- We can deduce that

$$K' = K^d(r, A', q)$$

- The last FONC implies an investment demand curve.

$$I(r, A', q, K)$$

- Investment is a decreasing function of r.
- Curve shifts out if $\uparrow A$.
- Investment demand would shift out in if $\downarrow K$.
- Investment is fundamentally forward-looking.
  - It depends importantly on expected future productivity.
- We express the demand as a function of the variables the firm takes as given.
  - We don't express it as a function of N'.
- What about labor supply?

Household

- Problem the same as before, except...
  - We are endogenizing labor/leisure decisions.
- We normalize the total endowment of time to 1 in each period.
  - This implies that leisure, l, is $1 - N$, where N is hours worked.
- We will assume that the consumption and leisure components of utility are separable.
- Households get utility from leisure via $v(1-N)$.
  - $v'(1-N) > 0$ and $v''(1-N) < 0$
- Lifetime utility given by:

$$\mathcal{U} = u(C) + v(1-N) + \beta\left[u(C) + v(1-N')\right]$$

Household: Budget Constraint

- Now we have to account for endogenous income.
  - Wages.
  - Dividends/profits from firms.
  - Households also pay taxes to the government.

$$C + S = wN - T + \Pi$$
$$C' = w'N' - T' + \Pi' + S(1+r)$$

- Intertemporal budget constraint:

$$C + \frac{C'}{1+r} = wN - T + \Pi + \frac{w'N' - T' + \Pi'}{1+r}$$

Household Problem

$$\max_{C,C',N,N'} \mathcal{U} = u(C) + v(1-N) + \beta\left[u(C) + v(1-N')\right]$$

Subject to:

$$C + \frac{C'}{1+r} = wN - T + \Pi + \frac{w'N' - T' + \Pi'}{1+r}$$

Or...

$$\max_{C',N,N'} \mathcal{U} = u\left[wN - T + \Pi + \frac{w'N' - T' + \Pi'}{1+r} - \frac{C'}{1+r}\right] + v(1-N) + \beta\left[u(C) + v(1-N')\right]$$

- Before solving this, let's get some intuition from a one-period model.

Optimal Decisions in a Static, One-Period Problem

$$\max_{C,N} u(C) + v(1-N)$$

Subject to:

$$C = wN + \Pi - T$$

Or...

$$\max_{N} u(wN + \Pi - T) + v(1-N)$$

- FOC:

$$v'(l) = wu'(C)$$

  - Relative price = MRS.
  - It is also the slope of the budget constraint.
  - This implicitly defines a labor supply curve.
- The optimal choices should satisfy the FOC and the BC.
- Remember (Π is D in GLS).

Analyzing the Optimal Decisions in a Static Problem

- Can analyze this in the indifference curve-budget line diagram.
  - An $\uparrow \Pi$ or $\downarrow T$ have a pure *income* effect.
    - Individual responds by consuming more goods and leisure.
- The question of greatest interest is how leisure behaves wrt to changes in wage.
- Offsetting effects:
  - *Income effect* lower labor supply.
  - *Substitution effect* raises labor supply.
- Draw *Frisch* labor supply curve: How does N vary with w, holding $u'(C)$ constant.
  - Must be upward sloping and shifts whenever C changes.
    - What happens if $\uparrow r$?
    - What happens if $\uparrow z'$?
    - What happens if $\downarrow g$ or $\downarrow g'$?

Household Problem

$$\max_{C',N,N'} \mathcal{U} = u\left[wN + \Pi - T + \frac{w'N' - T' + \Pi'}{1+r} - \frac{C'}{1+r}\right] + v(1-N)$$
$$+ \beta\left[u(C) + v(1-N')\right]$$

- Optimality conditions:

$$C: \quad u'(C) = \beta(1+r)u'(c')$$
$$N: \quad v'(1-N) = u'(C)w$$
$$N': \quad v'(1-N') = u'(C')w'$$

- Identical to the conditions from the static one-period problem.
  - In each period one has to decide optimal split between consumption goods and leisure —essentially a static decision.
  - Labor supply is obtained as before.
  - Many other variables will make the labor supply curve shift.

Government

- Same as before: G and G' chosen exogenously.
- Government's intertemporal budget constraint:

$$G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$$

- Ricardian equivalence holds:
  - Household behaves as though government balances budget every period.

Equilibrium Conditions

- Labor demand: $N^d = N(w, A, K)$.
- Labor supply: $N^s = N(w, \theta)$.
- Consumption: $C = C(Y - G, Y' - G', r)$.
- Investment: $I = I(r, A', q, K)$.
- Production function: $Y = AF(K, N)$.
- Market-clearing: $Y = C + I + G$

The Y^s Curve

The *aggregate supply curve*, denoted by Y^s, is the set of (r, Y) pairs consistent with household and firm optimization on the production side of the economy and with labor-market clearing.

- Basic idea behind its derivation:
  - Start with an initial r.
    - This determines a position of N^s through C.
  - Try a higher r.
    - This $\downarrow C$ and labor supply shifts out $\Rightarrow \uparrow N \Rightarrow \uparrow Y$
  - Hence, Y^s slopes up.
    - Higher r effectively makes people want to work more, and hence supply more output.

The Y^d Curve

The *aggregate demand curve*, denoted by Y^d, is the set of (r, Y) pairs consistent with household and firm optimization and $Y^d = Y$, where $Y^d = C + I + G$.

- Where did $Y^d = C + I + G$ come from?
  - This is because total demand, that comes from combining the period budget constraints of the different actors in the economy, can be expressed as:

$$Y^d = C + I + G$$

    - Standard accounting identity.
  - We have the optimal decision rules for each one of these items.
- Basic idea behind its derivation:
  - Use the expenditure line - 45 degree line diagram. Start with an r.
    - Determines position of expenditure line.
  - Increase r.
    - This causes expenditure line to shift down.
    - The intersection will be a lower point.
  - From there, it is easy to see that Y^d slopes down.

General Equilibrium

- General equilibrium requires that *all* markets clear.
- In this case we have effectively two markets:
  - Labor market ($N^s = N^d$).
  - Goods market ($Y^d = Y$).
- Market-clearing:
  - Labor market-clearing:
    - On Y^s curve.
  - Goods market-clearing:
    - On Y^d curve.
  - General Equilibrium:
    - On both curves.

Defining Equilibrium

Definition (Competitive Equilibrium)

A competitive equilibrium will be composed by a set of allocations —$C, C', N^{s'}, N^s, N^d, N^{d'}, I, \Pi$ and Π'— and prices —w, w' and r— such that:

1. *Given prices, households choose C, C', N^s, and $N^{s'}$ optimally.*
2. *Given prices, firm chose I, N^d, and $N^{d'}$ optimally.*
3. *The prices r, w, and w' are such that markets clear.*
4. *Profits are given by*

$$\Pi = zF(K, N) - wN - I$$

and

$$\Pi' = z'F\left[(1-\delta)K + I, N'\right] - w'N' + (1-\delta)\left[(1-\delta)K + I\right]$$

Graphical Analysis: Curve Shifts

- So we have five exogenous variables: z, z', G, G' and K.
- Shifts that may occur:
  - Labor demand:
    - Changes in z and K (more natural to think about a decrease in K, since K can't jump up).
  - Labor supply:
    - Anything that changes C other than things that affect current Y.
    - Changes in r, z', G, and G'.
  - Demand for goods:
    - Changes in z', G and G'.

Analyzing Effect of Changes in Exogenous Variables

- "Recipe" for thinking about how equilibrium responds to a change in an exogenous variable:
  1. Start in the labor market. Holding r and current income fixed, determine whether the change shifts N^s and/or N^d.
     - Decide whether N would change, for a given r. This will tell us whether Y^s shifts.
  2. Take the new value of N, bring it down, take it across and reflect it up.
  3. Figure out if Y^d shifts.
     - Y^d shifts whenever the quantity demanded would change for a given r.
  4. Combine the shifts in Y^d and Y^s to find the new equilibrium in the (r, Y) dimension.
  5. Figure out what happens with components of Y.
  6. Work back to labor markets to make quantities to line up.
     - The adjustment of r along Y^s occurs because of the shift of the N^s curve that occurs as r changes, so we need to go back to examine the final impact on the labor market (w and N).

Resolving Ambiguities

- Sometimes the curve shifts will produce ambiguities.
- Often, we can resolve them by doing some math.
- In particular, the labor market clearing condition is very useful:

$$v'(1 - N) = u'(C)zF_N(K, N)$$

- Under our assumptions, if neither z nor K moved, N and C must move in opposite directions.

Summary of Qualitative Effects

	$z \uparrow$	$z' \uparrow$	$G \uparrow$	$G' \uparrow$
Output (Y)	$+$	$+$	$+$	$+$
Hours (N)	?	$+$	$+$	$+$
Consumption (C)	$+$	?	-	-
Investment (I)	$+$	?	-	$+$
Real interest rate (r)	-	$+$	$+$	-
Real wage (w)	$+$	-	-	-

Taking Stock