Notes IV - Consumption-Saving Decisions

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Preliminaries

- In the Solow model we abstracted from the consumption/savings decision.
- Largest expenditure component in GDP (60–70%)
- Since this is macro, we will think about *aggregate* consumption (and savings).

Some of the Questions We're Interested in:

- How is the consumption/saving decision affected by:
  - Interest rate?
  - Expected income growth?
- Under what conditions would a reduction in income taxes stimulate consumption?
- How do changes in government expenditures affect household expenditures?
- Why is consumption less volatile than income?

Model

Overview of the Theoretical Framework

- Follow *representative agent* approach.
  - Modern macro features:
    - Agents are *forward-looking*.
    - Agents are rational and their behavior is optimizing.
    - Agents have a desire to smooth consumption across time.
- Lives for two periods: present and future.
  - Unrealistic, but we can analyze
    - Long-run decisions: life-cycle decisions.
    - Short-run decisions: this year vs. next year.
- Everything is real.
- There is no uncertainty.
- Exogenous income stream.

Throughout, we will discuss the limitations of these assumptions.

Budget Constraint

- *Endowment economy*.
  - Present and future:
    - y and y' or y_t and y_{t+1}.
- Each consumer pays a lump-sum tax, t and t'.
- Endowment can be consumed, c, or saved, s.
  - How can income be transferred between periods?
    - Riskless, interest-bearing asset.
    - Savings is a *stock*.
    - Saving is a *flow*, denoted by s.
  - Savings: $s > 0$.
  - Borrowing: $s < 0$.
  - Interest denoted by r.
  - We assume there are not limits on borrowing.

Intertemporal Budget Constraint

- By combining the budget constraints in both periods we have:

$$c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - t - \frac{t'}{1+r}$$

- This is a lifetime budget constraint.
  - What does it say?
- What is $\frac{1}{1+r}$?
  - Relative price of future consumption goods in terms of current consumption goods.
- We can label present **lifetime wealth** as *we*:

$$we = y - t + \frac{y'}{1+r} - \frac{t'}{1+r}$$

  - We can substitute *we* in the lifetime (or intertemporal) budget constraint.

Lifetime Budget Line

Shows combinations of c and c' that are consistent with the intertemporal budget constraint holding, given y, y', t, t', and r.

- In slope-intercept form we have:

$$c' = -(1+r)c + we(1+r)$$

- Graphically?
  - $we(1+r)$ is what could be consumed in $t+1$ if all the income today were saved.
  - we is the maximum consumption that could be achieved at t
    - By bringing all the future income to today.
  - E is the endowment point. No savings nor borrowing here.
- What about:
  - Points above E?
    - Consumer is a lender.
  - Points below E?
    - Consumer is a borrower.

Utility

- The "utility function" is a mapping:
  - $c \rightarrow u(c)$
- Utility properties:
  - $u'(c) \geq 0$
  - $u''(c) \leq 0$
    - Weakly concave function.
- Lifetime utility:

$$\mathcal{U} = u(c) + \beta u(c')$$

  - *Time-autonomous*: $u(c)$ doesn't change.
  - *Time-separable*: c' doesn't affect $u(c)$ or vice-versa.
- β is the *discount factor*.

Example of Utility Functions

- Some standard cases:
  - Linear:
$$u(c) = \theta c$$
  - Log:
$$u(c) = \ln c$$
  - Iso-elastic:
$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

Indifference Curve

- Remember life-time utility:

$$\mathcal{U} = u(c) + \beta u(c')$$

- We can represent the preferences as an indifference curve.
  - Combinations of c and c' that yield the same level of utility.
- We can total differentiate life-time utility and solve for the slope.
  - Slope = (-) ratio of marginal utilities at a point.
- If $u(c)$ is concave, the indifference curve has a convex shape.
  - Concave: MU is decreasing in consumption.
  - Agent would like to find a way to 'smooth' consumption.

Household's Problem

- We have this multivariate problem:

$$\max_{c,c'} \mathcal{U} = u(c) + \beta u(c')$$

subject to:

$$c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - t - \frac{t'}{1+r}$$

- We can transform that in an univariate problem:

$$\max_{c'} \mathcal{U} = u\left(y + \frac{y'}{1+r} - t - \frac{t'}{1+r} - \frac{c'}{1+r}\right) + \beta u(c')$$

- How to solve this?

Euler Equation

- The solution is given by:

$$\boxed{u'(c) = (1+r)\beta u'(c')}$$

  - *Euler Equation*.
- The **key** equation in Macroeconomics for understanding intertemporal trade-offs.
- Interpretation.
  - Agent makes the trade-off to the point where the loss and gain are equal.
- The importance of $\beta(1+r)$ for consumption growth.
  - If $\beta(1+r) = 1$: consumption is perfectly smoothed.
  - What if $\beta(1+r) > 1$?
  - What if $\beta(1+r) < 1$?

Euler Equation

- We can also rewrite the Euler as a *Marginal Rate of Substitution*.
- Relative price of present and future consumption.
- This is not a "consumption function".
- This is a condition characterizing an optimal consumption allocation.
  - It does not determine consumption on its own.

Graphical Solution

- From the budget constraint:
  - MRT.
- From the Euler equation:
  - MRS.
- Rearranging the Euler:

$$-\frac{u'(c)}{\beta u'(c')} = -(1+r)$$

  - This is a tangency condition.

Experiments: Changes in Income

- Increase in current income.
  - $\uparrow y$.
  - $\uparrow c, \uparrow c', \uparrow s$.
  - Marginal propensity to consume is less than 1.
  - Pure income effect.
- Increase in future income.
  - $\uparrow y'$.
  - $\uparrow c, \uparrow c', \uparrow s$.
    - $\Rightarrow$ Current consumption depends on future income.

Experiments: Changes in Interest Rates

- Increase in interest rate: $\uparrow r$.
  - Theoretically ambiguous what will happen with current consumption.
  - Depends whether household is a borrower or a lender.
  - Substitution effect is negative:
    - Price of current consumption increases.
    - Substitute away.
  - If initially a saver, income effect is positive.
    - However, which effect dominates depends on preferences and how much was being saved.
  - If initially a borrower, income effect is negative: $\downarrow c$.
    - Now current consumption decreases.
  - Unless otherwise noted, we assume substitution effect dominates:
    - $\downarrow c, \uparrow c', \uparrow s$.
    - Empirically plausible case.

Consumption Function

- What is the actual consumption bundle?
- Euler equation is not a consumption function.
- A consumption function endogenously explains consumption by incorporating things households take as given:
  - Income, taxes, and interest rates.
- Mathematically:

$$c = c(y, y', t, t', r)$$

- How to solve then for the actual bundle?
  1. Assume a functional form for the utility.
     - $u(c) = \ln c$.
  2. Solve for the Euler for c'.
  3. Plug (2) into the intertemporal budget constraint.
  4. Solve for c.

Experiments: Consumption Function

- Now we have:

$$c = \frac{1}{1+\beta} y + \frac{1}{(1+\beta)(1+r)} y' - \frac{1}{(1+\beta)} \left(t + \frac{t'}{1+r} \right)$$

- How is current consumption affected by an increase in:
  - Current income.
    - Marginal propensity to consume.
    - Bounded between 0 and 0.5.
  - Future income.
  - Interest rate.
    - Current consumption decreasing on interest rates.
    - Importance of the functional form assumed.

More Than Two Periods

Expanding the Model

- Why?
  - It will allow us to consider a wider range of issues.
- Let's look at a typical "life-cycle" consumption and income profiles.
  - Retire at age R, die at age T.
  - Assume $\beta(1+r) = 1$.
    - How should the path of consumption look like?
- Graphically?

What is the Household Problem Now?

$$\max_{\{c_t, s_t\}_{t=1}^{T}} \mathcal{U} = u(c_1) + \beta u(c_2) + \beta^2 u(c_3) + \cdots + \beta^{T-1} u(c_T)$$

Subject to:

$$\begin{aligned} c_1 + s_1 &= y_1 \\ c_2 + s_2 &= y_2 + (1+r)s_1 \\ &\cdots \\ c_t &= y_t + (1+r)s_{t-1} \end{aligned}$$

- We can also consolidate these budget constraints.
  - By repeatedly solving for s and substituting into the previous period budget constraint.
- Saving vs. Savings.
  - Flow vs. Stock.

Rewriting the Household Problem

Assuming retirement occurs at R:

$$\max_{\{c_t\}_{t=1}^{T}} \mathcal{U} = \sum_{t=1}^{T} \beta^{t-1} u(c_t)$$

Subject to:

$$\sum_{t=1}^{T} \frac{c_t}{(1+r)^{t-1}} = \sum_{t=1}^{R} \frac{y_t}{(1+r)^{t-1}}$$

- Consumption in consecutive periods are still linked by an Euler equation:

$$u'(c_{t+j}) = \beta(1+r)u'(c_{t+j+1}) \quad \text{for } j = 0, 1, \ldots, T-t-1$$

  - Intuition carries over:
    - It's all about trading marginal utilities at the same rate you can trade off resources.
- If $\beta(1+r) = 1 \Rightarrow c_t = c_{t+j}$.

Solving for Consumption

- In order to further simplify, let's assume that $r = 0$.
- Using the intertemporal budget, we can solve for consumption:

$$c = \frac{1}{T} \sum_{t=1}^{R} y_t$$

  - This assumes retirement at age R.
- The pattern of income doesn't matter.
  - The ability to borrow and save allows the consumer to decouple the path of consumption from the path of income.
- Same general idea holds even if $r \neq 0$

Temporary vs. Permanent Income Shocks

- What is the response of consumption to an increase of income that:
  1. Lasts one year.
  2. Is permanent.
- In the first case consumption will increase by a small amount.
- In the second case, consumption will increase by an amount equal to the full increase in income.

The marginal propensity to consume out of temporary increases in income is small, and the marginal propensity to consume out of permanent income increases is large.

- This idea was developed by Milton Friedman in 1957.
  - Stark contrast with the old Keynesian views that consumption was only a function of current income.

Permanent vs. Transitory Income Shocks: Numerical Example

- Suppose:
  - $T = 50$.
  - $R = 40$.
  - $y = \$10$ while working, $y = 0$ while retired.
    - What is total income?
    - What is average income?
- Consider a *temporary* increase in income:
  - Income increase by \$1 for one period (the first period).
    - How do income and consumption behave?
    - What happens with the extra income the period it increases?
- Now consider a *permanent* shock to income:
  - All incomes increase by \$1.
    - How do income and consumption behave?

Life Cycle / Permanent Income Hypothesis

- That's the generic name given to our theory of consumption.
- It has testable empirical predictions.
  1. Consumption ought to be forward-looking, depending not just on current income but future income as well.
  2. Consumption ought to react more to permanent changes in income than transitory changes in income.
  3. Current consumption should not react much to predictable changes in income that were anticipated in the past.

Empirical Evidence

- How well does our theory do?
  1. Social Security withholding.
  2. Retirement.
  3. Mortality.
  4. Tax rebates: 2001 vs. 2008.

Policy Implications: Tax Cuts and Consumer Spending

- Tax cuts are one means of "fiscal stimulus" that policy-makers occasionally use with the aim of stimulating consumer spending.
- We know it matters whether the tax cuts are permanent or temporary.
  - What is the difference?
- Yet President Bush in early 2008 enacted a temporary tax cut, whereby:
  - Single tax payers received a $600 check for a tax cut.
  - Married couples received $1200.
- Why could this be more effective than our current theory suggests?
  - We are coming back to this.

Anticipated vs. Unanticipated Income Shocks

- At t_0 the consumer discovers that at some point in the future, t_1, his income profile will increase.
  - What happens with consumption?
    - Consumption doesn't wait until t_1 to change.
    - It increases at t_0 because consumption is equal to average future income (assuming $r = 0$).
- Consider the 2008 tax refund.
  - Why rebate the check instead of announce that everyone will have their bill tax lowered by \$1200 when they file their taxes next year?
  - Hall (1978):
    - Any information about future income should affect consumption immediately.
    - Thus, current consumption is our best predictor of future consumption.
- This is just a theory.
  - In the data we observe an "excess sensitivity":
    - Current consumption seems to be more closely related to current income.

Ricardian Equivalence

Ricardian Equivalence: Government

- Let's study how government spending and the timing of taxes could affect the economy.
- Suppose government spends g and g' per person in the two periods.
- The resources to finance that spending are given by:
  1. Lump-sum taxes, t and t'.
  2. Borrowing: issuing bonds, b.
- What is the government budget constraint in each period?
- As before, let's obtain the government's *intertemporal budget constraint*.
  - This implies that the government should live within its means.
- What about households?

Ricardian Equivalence

A change in the timing of taxes has no real effect in the economy.

Example:

- While holding spending fixed, government cuts taxes by x.
  - It must raise borrowing by x.
  - Future taxes must rise by $x(1+r)$ in order to pay the debt.
  - What happens with household budget constraint?
    - Nada.
    - Therefore, nothing happens to c and c'.
  - However household increases saving.
    - This is used to pay higher future taxes.
- There is an *equivalence* between the financing of government spending:
  - Collecting taxes now.
  - Issuing debt and raising future taxes to pay for the increase in debt.

Ricardian Equivalence and Fiscal Stimulus

- This means that fiscal stimulus via tax cuts cannot be effective.
  - They will not stimulate consumption.
- What about fiscal stimulus via government spending.
  - Government increases demand for goods by Δg.
  - Households will decrease c by less than the increase in g.
  - Overall demand increases.
- Increase in government borrowing exceeds the increase in household savings.
  - Where do funds come from?
    - We will return to this when we talk about the determination of interest rates.

Why Might Ricardian Equivalence Not Hold?

- Could it be that households aren't really life-cicle/PIH decision makers?
  - That is, are they myopic?
- What about an intergenerational context?
  - Do current generations accumulate savings to account for future tax burden?
  - Do people care about future generations?
    - If so, we shouldn't worry about the current looming shortfall in social security.
- Why we might not observe a Ricardian Equivalence:
  1. Taxes aren't really lump sum.
     - If taxes are distortionary, they will have real effects.
  2. Binding borrowing or liquidity constraints.
     - Perhaps households are consuming less that they would if they could borrow more.

Reconciling Theory and Data

Binding Liquidity Constraints

- Household is restricted in their ability to borrow.
  - Mathematically:
$$s \geq 0 \quad \text{or} \quad c \leq y$$
  - This could also be achieved by distinguishing interest rates on lending and borrowing.
- Increase in future income, $\uparrow y'$:
  - c does not react, c' "over-reacts" relative to baseline theory.
- Increase in current income, $\uparrow y$:
  - c "over-reacts" to transitory change in income.

Reconciling Theory and Data

- Borrowing constraints resolve the "excess sensitivity puzzle".
  - They make consumption much more responsive to temporary changes in income.
  - It also show how consumption could respond to an anticipated change in income.
    - Consumption cannot respond today when news of higher future income arrives.
    - It has to wait until higher income is actually received.
- Policy implications:
  - "Targeted" tax cuts.

Borrowing Constraints and Ricardian Equivalence

- Consumers may now respond to a tax cut even though they know taxes will rise.
- How do these things add up?
  - Households are consuming more and saving less.
  - Government needs to borrow more.
- We need to start thinking about equilibrium determination of interest rates.
  - Will the interest rate rise?

What's next?