Notes I:
Introduction and Useful Mathematical Concepts

Julio Garín

Intermediate Macroeconomics

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“Advice”

- Attendance.
- Behavior in class.
  - Why classroom etiquette is important.
  - You should read the syllabus.
    - However, some things may change (outline and dates, specially).
- How is this class going to be taught?
  - Book are important, but the single most important thing is to follow the notes and the material covered in class.
    - It is your responsibility to be aware of what we covered.
- Assignments will be a combination of:
  1. Homework.
  2. Reading Assignments.
  3. Tests.
- Goal is to give you solid foundations.

How to Prepare and Work For This Class?

- Don't fall behind.
  - You have plenty of resources.
- Read, print the notes, and complete them in class.
  - Seriously.
- Start the homework ASAP.
- Read the NYT, WSJ, and The Economist.
  - Share the material that you think is interesting.
- Work in groups while possible.

Don't Take Shortcuts

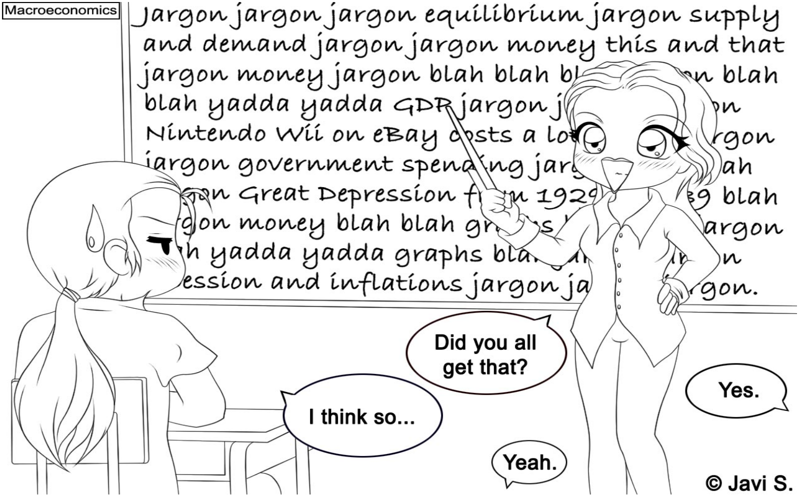
- Read Claremont McKenna's Statement of Academic Integrity.

Don't take nothing from nobody.
– *Miley Cyrus*

Basics

- About homework and office hours.
- About grading.
  - Here, a fixed scale doesn't make sense.

What if You Don't Understand Something?



Introduction

- Distinction between micro and macroeconomics is not that clear.
  - Economics is micro. Macro just studies these issues at the aggregate level.
  - The emphasis is on the intertemporal nature of economic decisions.
- *Some* of the questions we are going to address:
  - What are the main determinants of economic growth?
    - Can policymakers affect the growth rate?
  - Why does the economy experience periodic deviations from its growth path–i.e. business cycles?
    - What are the implications?
    - Can policymakers dampen the fluctuations? If so, should they?
    - Is fiscal policy or monetary policy more effective?
  - WTH happened in 2007–2009?

Intermediate Macroeconomics

- Principles provided you with concepts, definitions, and graphical arguments.
  - We will analyze *formal models*.
- Why?
  - Real world is very complex.
  - Consider an increase in government spending.
    - This should increase demand for goods.
    - What if households anticipate future tax increase?
    - They will reduce the demand for goods!
    - Which of these two forces dominates?
  - Is it something "too much" or "too little".
  - We like to think about "what if?".
- A model allows us to formalize these ideas and helps us to answer these questions.

Okay, got it, but do we really need more math?

- What do we want?
  1. We want our models to be based on *optimizing behavior* by individuals.
  2. We also want our models to be *equilibrium* models –decisions are consistent with what is going on.
  3. Since many economic decisions involve intertemporal tradeoffs, we need to think about *dynamic* models.
- This deeper, more formal approach will require that we utilize more math and it is the basis of modern macroeconomics.
- Hence, the answer is yes.

That said, it is true that modern macroeconomics uses mathematics and statistics to understand behavior in situations where there is uncertainty about how the future will unfold from the past. But a rule of thumb is that the more dynamic, uncertain and ambiguous is the economic environment that you seek to model, the more you are going to have to roll up your sleeves, and learn and use some math. That's life.
– *Thomas Sargent*, 2011 Nobel Prize Winner

Math Background

Outline of Topics

- Notation.
- Exponents and logs.
- Growth rates.
- Calculus.
- Optimization.
- Summations.

Notation

- We have two types of variables.
  1. Exogenous.
  2. Endogenous.
- We also have *parameters*: fixed values governing mathematical relationships.
- Time is discrete.
- We are going to use the summation notation.

Properties of Exponents and Logs

- Exponents.
  - $x^a x^b =$
  - $\frac{x^a}{x^b} =$
  - $(x^a)^b =$

1. Product rule: $\ln(\alpha\beta) =$
2. Quotient rule: $\ln(\alpha/\beta) =$
3. Power rule: $\ln(x^\alpha) =$

- Remember: $\ln = \log_e$.
  - Since $log_x x = 1$, what is $\ln(e^\alpha)$ equal to?

More on Properties and Growth Rate

- Growth rate: $g_t^x =$
- Fun facts:
  - $\ln(1+\alpha) \approx \alpha$ and
  - $\exp(\alpha) \approx 1+\alpha$ for small α.
- $g_t^x \approx \ln x_t - \ln x_{t-1}$

Calculus: Notation and Preliminaries

- Consider a function $y = f(x)$.
- First derivative: how y changes as x changes.
  - Derivative is itself a function.
  - Notation:
    - $\frac{dy}{dx}$, $f'(x)$, and $f_x(x)$.
- Second derivative: derivative of a derivative.
  - Notation:
    - $\frac{dy^2}{d^2x}$, $f''(x)$, and $f_{xx}(x)$.
- Don't confuse a derivative with a derivative evaluated at a point.
  - $f'(x) \neq f'(x_o)$.

Calculus: Derivative Rules

- Constant.
- Power, logs, and exponents.
- Derivative of a sum.
- Product rule.
- Chain Rule.

Derivative Rules: Examples

1. $f(x) = x^{\alpha}$.
2. $f(z) = \beta z^{(1-\alpha)}$.
3. $f(c) = \gamma \ln(c)$.
4. $f(k) = (k^{\alpha}) \left(\beta k^{1-\gamma}\right)$.
5. $f(g(c)) = (c^2 - c^3)^{\alpha}$.
6. $f(g(x)) = (x^{\alpha})^{\gamma}$.

Calculus: Multivariate Derivatives

- Let's assume there are two variables: $y = f(x, z)$.
- *Partial derivative* is change in x for a change in x (or z), holding z (or x) constant.
- Notation:
  - First derivative: $\frac{\partial y}{\partial x} \equiv f_x(x, z)$.
  - Second derivative: $\frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right) \equiv \frac{\partial^2 y}{\partial x^2} \equiv f_{xx}(x, z)$.
  - Cross derivative: $\frac{\partial}{\partial x}\left(\frac{\partial y}{\partial z}\right) \equiv \frac{\partial^2 y}{\partial x \partial z} \equiv f_{xz}(x, z)$.

Multivariate Derivatives: Examples

- For the following functional forms:
  1. $f(x, z) = \ln x + z^{\alpha}$.
  2. $f(k, n) = k^{\alpha} n^{(1-\alpha)}$.
  3. $f(x, z) = \beta x^{\alpha} + 3xz + \gamma z^{\theta}$.
- Calculate, for each function:
  1. First and second partial derivatives.
  2. Find the cross partial derivatives.

Calculus: Optimization

- Pick x to either maximize or minimize $f(x)$.
- First order condition (FOC):
  - x^* needs to satisfy $f'(x^*) = 0$
- Second order condition (SOC):
  - Sign of $f''(x^*)$ tells you whether you have a maximum or a minimum.
- Multivariate optimization works in the same way:
  - FOCs set partial derivatives with respect to (wrt) each choice variable equal to zero.

Optimization: Examples

- For the following functions:
  1. $f(x) = x^2$.
  2. $f(c) = \ln c - 2c$.

  Calculate:

  i FOCs.
  ii SOCs.
  iii Is this a maximum or a minimum?

Calculus: Constrained Optimization

- The problem now is to optimize $f(x, z)$ subject to an inequality constraint that x and z must satisfy.
- Our approach:
  1. Assume constraint holds with equality.
  2. Eliminate one of the choice variables.
  3. Perform an unconstrained optimization.

Constrained Optimization: Example

Let's solve this problem:

$$\max_{x,z} U = \ln x + \ln z$$

subject to

$$p_x x + p_z z \leq y.$$

Summation

- Sigma notation ($\sum_{\text{from}}^{\text{to}}$).
- General manipulations.
  1. Constant.
  2. Additivity.
  3. Linearity.
  4. Constant multiple.