Borrowing Constraints, Collateral Fluctuations, and the Labor Market*

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Abstract
This paper studies the effects of changes in collateral requirements on the cyclical properties of unemployment and job creation. I develop a general equilibrium model in which labor market frictions prevent the costless adjustment of employment. Financial frictions arise from an imperfect enforcement contract. An environment in which borrowing limits are linked to the firm’s physical capital stock can quantitatively account for the sluggish response of labor market variables to productivity shocks. I find that fluctuations in those variables are mainly driven by changes in financial conditions. The model can explain 75% of the variation in job creation observed in the data, and it can also account for the persistent reduction in both output and leverage that follows a contraction in credit availability.

JEL Classification: E24; E27; E32; E44; J63; J64.

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1 Introduction

The 2008 financial crisis has highlighted the need for a better understanding of the extent to which financial frictions can affect macroeconomic aggregates. The fact that a tightening of credit conditions was followed by a substantial increase in unemployment rates suggests that understanding how fluctuations in job creation are affected by changes in the availability of credit for firms not only constitutes an important theoretical exercise, but also an essential policy matter. Nevertheless, the literature has yet to focus on how exogenous changes in collateral requirements affect both labor markets and the hiring decision of firms in a fully stochastic and dynamic general equilibrium environment. Motivated by this, I develop a general equilibrium environment, with frictions in both labor and financial markets to quantitatively assess the importance of changes in credit availability in accounting for the cyclical dynamics in unemployment and vacancy creation.

Can a framework with these characteristics bring us closer to explaining the volatility observed in labor market variables? Moreover, what fraction of these fluctuations can be explained by variations in credit conditions? What are the implications of a sudden increase in credit tightness in an economy in which firms are financially constrained and there are costs and frictions associated with hiring workers? The framework presented is suited to answer these questions. I show that an environment in which borrowing limits are linked to the firm’s physical capital stock can quantitatively account for the sluggish response of labor market variables to productivity shocks. In addition, I find that fluctuations in those variables are mainly driven by changes in financial conditions. Quantitatively, the model can account for 75% of the variations in job creation observed in the data.

I model financial frictions as arising from a contract with imperfect enforcement, in the spirit of Kiyotaki and Moore (1997). A firm’s ability to borrow is constrained to be less than a fraction of its collateralizable assets. The collateral consists of the stock of capital owned by the firm at the beginning of the period. Subjecting the firm to this type of collateral constraint has the advantage that it provides a direct link between collateral requirements and asset prices, which some authors argue played an important role in the 2008 financial crisis. Recent studies have emphasized the importance of financial shocks in generating fluctuations in macroeconomic aggregates. I borrow from these studies and introduce exogenous variations in collateral requirements, which I refer to as ‘credit shocks’. These variations are meant to capture the uncertainty in credit conditions faced by firms. Together with the borrowing constraint, these disturbances capture the fact that credit tightness fluctuates over

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1See for instance Geanakoplos (2009) and Krishnamurthy (2010).
2Important examples are Jermann and Quadrini (2012) and Liu et al. (2013).
time. While those fluctuations may arise as optimal responses to other changes in the economy, modelling the endogenous determination of financial conditions is beyond the scope of this paper. The approach here is to draw out the implications of the deteriorating credit conditions.

The model presented in Section 2 has two types of agents. Households supply labor and funds. Capitalists do not supply labor but rather own the firms (and thus own the capital stock). In the model, firms can finance their operations through the use of debt (issued to households) or equity and are subject to a cash flow mismatch that requires them to take intra-period loans. As the possibility of default is assumed to arise at the end of the period, both inter- and intra-period loans are subject to the collateral requirement. A negative credit shock, i.e. a tightening of credit conditions, reduces the amount the firm can borrow against its collateral.

The labor market is modeled as a search and matching environment. This introduces frictions that make hiring a costly process. It follows that periods in which the availability of credit is tight are also periods in which it is relatively more costly to post vacancies. As firms are financially constrained, a contraction in credit directly affects the ability of firms to create jobs. Following a tightening of credit, the reduction in vacancy creation is significant. Since it takes several quarters until hiring can catch up with the worker separations that occur every period, the effects of financial conditions on unemployment are persistent.

Working capital requirements induce firms to cut job creation following a tightening in credit conditions and this effect is amplified by the way wages are determined, i.e. through a bargaining problem. Unlike standard labor search models, the bargaining position of firms is not constant but depends on credit market conditions. A contraction in credit endogenously improves the bargaining position of firms by increasing the sensitivity of the firm’s surplus to changes in wages. This implies that, relative to a standard search and matching framework, small changes in wages generate larger movements in labor market variables. In contrast with recent studies that also address the effects of credit shocks, a contraction in credit availability is accompanied by an increase in the effective bargaining position of firms. This allows the model to generate declining wages during periods of reduced job creation. Furthermore, the decrease in borrowing that follows a contraction in credit generates deleveraging consistent with the evidence that these are periods in which firms reduce their level of debt relative to their level of production (Reinhart and Reinhart (2010)).

The presence of credit constraints can quantitatively account for the empirical sluggishness in the response of employment and labor market tightness (the ratio of vacancies to unemployment, \( v/u \)) that follows a productivity shock. Following a positive productivity shock, firms prioritize investment in the asset used as collateral: capital. Relative to a model
without financial frictions, the firm’s preference for relaxing the constraint (by increasing its capital stock) reduces the immediate increase in vacancy posting. This generates a more protracted and persistent response of employment and labor market tightness. Quantitatively, the $v/u$ ratio continues to increase for three consecutive quarters after the shock and more than 60% of its total increase occurs in periods that follow the shock.

In Section 3 I carry out a quantitative exercise in which the mean and the standard deviation of the process that governs the credit shock in the model are calibrated to match the empirical features of the debt-to-GDP ratio. The model is successful in producing volatility in the extensive margin, generating an unemployment rate that is almost four times as volatile as GDP. Although this is still less than the relative volatility of unemployment observed in the data (which, with the detrending used, it is almost seven times higher than the volatility of GDP), it represents a significant improvement relative to standard models that can only generate an unemployment rate whose volatility is on par with that of output (see Shimer (2005)).

The model can account for more than 75% of the variation observed in vacancies and roughly 40% of the fluctuations observed in labor market tightness. I find that while productivity shocks are important in generating movements in output and investment, credit shocks are responsible for an important share of the fluctuations observed in labor market variables. This result is mainly driven by a low sensitivity of wages with respect to credit shocks, so that changes in financial conditions do not fully translate into changes in wages (in contrast with Shimer (2005), for example), but instead generate movements along the extensive margin. Put differently, in contrast with productivity shocks, the adjustments that follow changes in credit conditions are mainly through quantities and not prices. Negative productivity shocks decrease the incentives of the firm to invest and to post vacancies, therefore reducing the degree of credit tightness faced by firms. However, a negative financial shock generates the opposite result and considerably increases credit tightness. Since the firm’s value of having an extra worker depends directly on credit tightness, the reduction in vacancy posting that follows a contraction in credit availability is significant. In this way, fluctuations in collateral requirements are important in explaining business cycle movements in labor market variables.

This paper adds to a growing literature that seeks to understand the effects of exogenous and unexpected changes in the availability of credit on macroeconomic aggregates. Recent papers have explored the extent to which these variations in collateral requirements influence the dynamic response of economic variables. Notable examples being Jermann and Quadrini (2012) and Liu et al. (2013). Despite the important insights provided by these studies, they are not suited to analyze variations in the extensive margin, which are responsible
for most of the fluctuations in total hours (Rogerson and Shimer (2011)). An exception constitutes Monacelli et al. (2011), who seeks to capture evidence found in the corporate finance literature that suggests that the level of a firm’s borrowing affects its bargaining position. In their paper, a contractions in credit, by allowing workers to extract higher wages, affects the firms’ willingness to create jobs. While Monacelli et al. (2011)’s environment does not feature physical capital accumulation and focuses on another channel that influences job creation, our papers are related. I also depart from perfect labor markets and incorporate search frictions à la Pissarides (1985) but using the representative household framework developed in Merz (1995) and Andolfatto (1996).

The paper is also connected with some studies that have sought to resolve what has become known as the ‘Shimer puzzle,’ regarding the inability of the Pissarides (1987) matching model to explain the fluctuations in labor market variables observed in the data (Shimer (2005), Hall (2005), and Hagedorn and Manovskii (2008)). Petrosky-Nadeau (2014), addresses the puzzle in a model with financial frictions. His model differs from the one presented here in that he incorporates financial frictions that arise from a problem of asymmetric information as in Carlstrom and Fuerst (1997). The asymmetric information between financial intermediaries and firms implies that firms are able to access funds as long as they commit to the payment of the required interest rate. Put differently, in his environment, a worsening of credit conditions takes the form of an increase in lending rates. In order to study the role of exogenous changes in the availability of credit given by changes in collateral requirements, I depart from these studies by modelling financial frictions as arising from a contract with imperfect enforcement. Hence, in this paper, negative credit shocks are akin to a contraction in the quantity of available credit. Next, I introduce the model developed in the paper.

2 The Model Economy

The baseline model has two types of agents: workers and capitalists/entrepreneurs. These agents interact in an environment characterized by frictions in the labor market and in the financial sector.

Financial frictions are assumed to result from a contract with imperfect enforcement as in Kiyotaki and Moore (1997). This implies that firms face collateral requirements that limit their ability to borrow. I assume market segmentation in the sense that workers cannot hold

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3Focusing on productivity shocks and with a dynamic extension of Wasmer and Weil (2004) in which search frictions are present in both financial and labor markets, Petrosky-Nadeau and Wasmer (2013) show that the introduction of financial frictions can considerably amplify the responses of labor market variables. Using the same environment to model financial frictions, Chugh (2013) incorporates physical capital accumulation and brings the study of Petrosky-Nadeau (2014) closer to the DSGE search literature.
shares in the firms, which are exclusively owned by capitalists, and the only asset that is available to workers is a one-period riskless bond issued by capitalists. There is a continuum of firms that hire labor and accumulate physical capital in order to produce a homogeneous good and transfer dividends to its owners, the capitalists (I use the terms firms and managers interchangeably throughout the paper).

With the underlying assumption that it is costly for firms to both find and hire workers, labor market frictions are introduced in the spirit of Pissarides (1985) and, as is common in the literature that focuses on labor search frictions, wages are determined as the solution to a bargaining problem between workers and firms. As is standard in DSGE search models since the seminal work of Merz (1995) and Andolfatto (1996), it is assumed that each household consists of a continuum of measure one of family members. Within each household there is perfect risk-sharing so that, despite their individual labor market status, consumption is equalized across members.

Before describing the characteristics of financial markets and labor markets, I present the main features of households, capitalists, and firms. Then, I introduce the optimization problems faced by households and firms.

2.1 Households

There is a representative household in the economy composed of a continuum of measure one of family members. The household pools the income of all its members and allocates consumption in order to maximize utility. In doing so it equalizes the marginal utility of consumption across all individuals, independent of their labor market status (Andolfatto (1996)). Assuming a utility function with separability between consumption and leisure and perfect risk sharing, all individuals will have the same level of consumption. Households maximize lifetime utility,

$$E_t \sum_{j=0}^{\infty} \beta^j_h [u(c_{t+j}) - \varphi n_{h,t+j}]$$

where $c$ is household consumption, $\varphi$ is the disutility of work, $n_{h,t}$ is the measure of individuals who are employed at time $t$, and $\beta_h$ is the household’s discount factor.

Employed individuals earn wages, $w_t$, and unemployed members receive unemployment benefits, $s$. The latter are financed through the payment of a lump-sum tax, $T_t$. Households have access to financial markets. Specifically, they have the possibility of smoothing consumption across periods by purchasing a one-period riskless bond, $a_t$. The household’s
flow of funds constraint can be written as,

\[ c_t + \frac{a_{t+1}}{R_t} + T_t \leq w_t n_{h,t} + a_t + (1 - n_{h,t}) s. \]

### 2.2 Capitalists and Firms

As in Perri and Quadrini (2011), capitalists derive utility from the consumption financed out of dividends obtained from their ownership of firms. Those agents are assumed to be risk-averse and hence they would like to avoid fluctuations in their consumption. Without this assumption, credit shocks would be severely dammed as the firm would be able to costlessly adjust the sources of funds, considerably reducing the effects of financial frictions. The important element here is that dividend adjustment is costly. An alternative specification would be to assume, as in Jermann and Quadrini (2012), that firms are subject to costs associated with adjusting dividend payments.

Capitalists have access to the financial sector only through the firm. This implies that they consume all the dividends received, \( d_t \), and their lifetime expected utility at period \( t \) is \( E_t \sum_{j=0}^{\infty} \beta_t^{t+j} u(d_{t+j}) \) where \( \beta_t \) is their discount factor. Consequently, their stochastic discount factor is given by \( \Lambda_t = \beta_t^{t} u'(d_{t+j}) / u'(d_t) \). I assume that capitalists are relatively more impatient than households, i.e. \( \beta_h > \beta_c \). This impedes firms from accumulating enough assets in order to avoid the constraint and, as will be shown later, implies that in steady state the borrowing constraint is binding. This assumption is common in the literature and its importance will become clear once I discuss the borrowing constraint faced by firms.

Firms maximize the expected future stream of dividends, discounted by the stochastic discount factor of capitalists (the firms owners). The firm’s objective can be written as

\[
\max E_t \sum_{j=0}^{\infty} \Lambda_t^{t+j} d_{t+j}
\]

Firms can borrow via a one-period riskless bond, \( b_{t+1} \), whose gross interest rate is \( R_t \). They produce a homogenous good using a Cobb-Douglas technology with capital, \( k_t \), and labor, \( n_{c,t} \), as inputs. Specifically,

\[ y_t = z_t k_t^\alpha n_{c,t}^{1-\alpha} \]

where \( z_t \) is the level of total factor productivity (TFP). This level is common to all firms and evolves according to \( \ln z_t = \rho z \ln z_{t-1} + \epsilon_{z,t} \) with \( \epsilon_{z,t} \sim N(0, \sigma_z) \). Given that the model does not feature idiosyncratic shocks, I focus on a symmetric equilibrium where all the firms behave in the same way.
2.3 Labor Markets

Job matches are obtained from a Cobb-Douglas matching technology \( m(U_t, v_t) = \nu U_t^\gamma v_t^{1-\gamma} \), where \( \nu \) reflects the efficiency of the matching process, \( U_t \) and \( v_t \) are unemployment and vacancies posted by firms in period \( t \), respectively, and \( \gamma \) is the elasticity of matches with respect to unemployment. The probability that a firm fills a vacancy, i.e. the job filling rate, is given by \( \mu(\theta_t) \equiv m(U_t, v_t)/v_t = \nu \theta_t^{-\gamma} \), where \( \theta_t \equiv v_t/U_t \) represents the labor market tightness. Equivalently, \( f(\theta_t) \equiv m(U_t, v_t)/u_t = \nu \theta_t^{1-\gamma} \) is the job finding rate.

At the beginning of every period, before matches are realized, a fraction \( x \) of all employed workers are exogenously separated from firms. I follow Ravenna and Walsh (2008) and assume that workers that become separated at the beginning of the period have a probability \( f(\theta_t) \) of finding a job within the period. Therefore, employment is given by the previous period’s workers that were not separated plus all the employees matched this period. Formally,

\[
n_t = (1-x)n_{t-1} + m(U_t, v_t). \tag{1}
\]

The fraction of unemployed workers that are eligible to find a match is given by \( U_t = 1 - (1-x)n_{t-1} \). In the quantitative exercises I focus on unemployment at the end of the period, \( u_t \), which is defined as \( u_t = 1 - n_t \).

2.4 Financial Markets

In addition to the intertemporal loan, \( b_{t+1} \), firms are required, due to a cash-flow mismatch, to raise funds via intra-period loans, \( l_t \). Therefore, and following recent literature, I assume that firms face working-capital needs that have to be satisfied by obtaining an intra-period loan to cover dividends as well as the total cost of production.\(^4\) As in Jermann and Quadrini (2012), payments to workers, \( w_t n_{c,t} \), shareholders, \( d_t \), investment expenditures, \( i_t \), expenses related with creating a vacancy and hiring a worker, \( \psi(v_t) \) and \( \nu v_t \mu(\theta_t) \) respectively, and current debt net of new issue, \( b_t - b_{t+1}/R_t \), have to be made before the realization of revenues. Therefore, the intra-period loan can be written as,

\[
l_t = d_t + w_t n_{c,t} + \nu v_t \mu(\theta_t) + \psi(v_t) + i_t + b_t - \frac{b_{t+1}}{R_t}.
\]

Note that \( l_t = y_t \) so one can think of \( l_t \) as being the liquid funds that the firm possesses.

Financial frictions emerge due to the presence of costly contract enforcement. Therefore, the possibility of default implies that firms are subject to a collateral requirement. As in

\(^4\)See for instance Neumeyer and Perri (2005) and, recently, Perri and Quadrini (2011), and Abo-Zaid (2015) for other studies that incorporate working capital needs in the modeling of the firm.
Kiyotaki and Moore (1997), the borrowing of the firm is limited by a fraction $\eta_t$ of the value of the physical capital stock held by the firm at time $t$. This value is given by $q_{k,t}k_t$, where $q_{k,t}$ represents the (marginal) Tobin’s $Q$. I follow Liu et al. (2013) by interpreting $\eta_t$ as an exogenous “collateral shock,” which reflects the uncertainty in the tightness of the credit market. From the lender’s perspective, $\eta_t$ captures the uncertainty with respect to the liquidation value of the firm and its dynamics are represented by the following stochastic process:

$$\ln \eta_t = (1 - \rho_\eta) \ln \bar{\eta} + \rho_\eta \ln \eta_{t-1} + \epsilon_{\eta,t}$$

with $\epsilon_{\eta,t} \sim \mathcal{N}(0, \sigma_\eta)$, where $\bar{\eta}$ is the mean value of the process and $\rho_\eta$ is its persistence. Formally, the constraint faced by the firm is given by

$$l_t + \frac{b_{t+1}}{R_t} \leq \eta_t q_{k,t}k_t.$$  \tag{2}

In Appendix D, I provide the derivation of Equation (2) as the outcome of a an incentive compatible contract between financial intermediaries and firms.

### 2.5 Household’s Optimization Problem

The household takes the job finding rate as given, hence perceiving employment as evolving according to

$$n_{h,t} = (1 - x)n_{h,t-1} + f(\theta_t)U_{t-1}.$$  

Every period households choose the level of consumption, $c_t$, and the number of riskless bonds in order to maximize expected discounted utility over consumption and leisure. Letting $\omega_t^h = \{n_{h,t-1}, a_t\}$ be the vector of individual states for households and $\Omega_t = \{k_t, n_{t-1}; z_{t-1}, \eta_{t-1}\}$ be the vector of aggregate states, where $n_{t-1}$ is period $t - 1$ employment, the problem can be summarized as

$$H(\omega_t^h; \Omega_t) = \max_{\{c_t, a_{t+1}\}} \left\{ u(c_t) - \varphi n_{h,t} + \beta_h E_t H(\omega_{t+1}^h; \Omega_{t+1}) \right\}$$  \tag{3}

subject to

$$c_t + \frac{a_{t+1}}{R_t} + T_t = w_t n_{h,t} + a_t + u_t s$$  \tag{4}

and the laws of motion for $z_t$ and $\eta_t$.

The first-order conditions with respect to consumption and to riskless assets give the
usual consumption Euler equation

\[ \frac{1}{R_t} = \beta_h \mathbb{E}_t \frac{u'(c_{t+1})}{u'(c_t)}. \]  

(5)

I denote the discounted intertemporal marginal rate of substitution, which at the optimum will be equal to the household’s stochastic discount factor, as as \( \Lambda_{t+j|t}^h = \beta_h \frac{u'(c_{t+j})}{u'(c_t)}. \)

2.6 Firm’s Optimization Problem

Managers determine the measure of workers that will be active in the production process at period \( t, n_{c,t} \), by posting vacancies, \( v_t \). The costs associated with the latter are given by the function \( \psi(v_t) \) which can be linear, concave, or convex depending on whether the marginal cost of vacancy postings are constant, diminishing, or increasing. Since the probability that a firm fills a vacancy is given by \( \mu(\theta_t) \), the total hiring of the firm at time \( t \) is given by \( v_t \mu(\theta_t) \).

In order to capture that the cost of posting vacancies is not the only one faced by firms, it is assumed that for each successful match firms have to pay a fixed cost, \( \iota \), as a ‘start-up’ cost. Start up costs represent the resources that must be allocated to incorporate newly hired workers into the production process, e.g. training costs. From the firm’s perspective, the evolution of the number of workers given by

\[ n_{c,t} = (1-x)n_{c,t-1} + v_t \mu(\theta_t). \]  

(6)

At time \( t \), firms also decide the level of capital that is going to be used in the production process at \( t+1 \). The law of motion for capital accumulation is

\[ k_{t+1} = (1-\delta)k_t + \Phi \left( \frac{i_t}{k_t} \right) k_t \]  

(7)

where \( \delta \) is the depreciation rate, and \( \Phi(i_t/k_t) \) is a function that captures costs associated with adjusting the capital stock. The budget constraint faced by firms is given by

\[ z_t k_t^{\alpha} c_{c,t}^{1-\alpha} + \frac{b_{t+1}}{R_t} = d_t + wtn_{c,t} + i_t + \nu \mu(\theta_t) + \psi(v_t) + b_t. \]  

(8)

Firms then maximize capitalists’ wealth by choosing dividends, the number of vacancies to post, new debt to issue, next period’s capital stock, and level of investment. Defining \( \omega_t^e = \{k_t, n_{c,t-1}, b_t\} \), the problem can be expressed recursively as

\[ J(\omega_t^e; \Omega_t) = \max_{\{d_t, v_t, i_t, k_{t+1}, b_{t+1}\}} \{d_t + \mathbb{E}_t \Lambda_{t+1|t}^e \{J(\omega_{t+1}^e; \Omega_{t+1})\} \} \]  

(9)
subject to Equations (2), (6), (7), and (8). When optimizing, the individual firm takes as
given the probability that a vacancy will be filled, \( \mu(\theta_t) \), the gross interest rate, \( R_t \), the
stochastic discount factor of capitalists, \( \Lambda_{c|t+1}^{c} \), and the wage, \( w_t \). Letting \( \mu_{c,t}, \mu_{k,t}, \mu_{b,t}, \) and
\( \mu_{e,t} \) denote, respectively, the multipliers on the budget constraint, (8), the law of motion for
capital, (7), the borrowing constraint, (2), and the law of motion for employment, (6), the
first order necessary conditions for the firm are derived in Appendix A.2.

2.6.1 Job Creation

The job creation equation is convenient to analyze how financial frictions can affect the
incentives of firms to create jobs. Before doing so, it may be useful to analyze the marginal
value of having an extra worker in the firm, \( J_{n,t} \), that is obtained by taking the derivative of
the firm’s value function, \( J \), with respect to employment:

\[
J_{n,t} = \left( 1 - \mu_{b,t} \right) \left( 1 - \alpha \right) z_t \left( \frac{k_t}{n_{c,t}} \right)^{\alpha} - w_t \right] + (1 - x)\mathbb{E}_t \Lambda_{t+1}^{c} \tilde{J}_{n,t+1}.
\]

The term in square brackets corresponds to the net return of having an extra worker in the
firm, while the second term is the present discounted value of the hired worker. Note that,
relative to a case with perfect financial markets (\( \mu_b = 0 \)) the presence of credit constraints
affects the marginal value of having an extra worker.\(^5\) Intuitively, an increase in collateral
requirements makes the firm more credit constrained (increase \( \mu_b \)), therefore reducing the
net value of an extra worker by making wages relatively more costly. Put differently, credit
constraints reduce the marginal benefit of hiring a worker by a factor of \( \mu_{b,t}/(1 - \mu_{b,t}) \).

In Appendix A.2 the job creation equation is derived. However, to gain more intuition
from a comparison with a case with perfect financial markets, it may be useful to express the
equation as:

\[
\frac{\mu(\theta_t) + \psi'(v_t)}{\mu(\theta_t)} (1 + \tau_t) = F_{n,t} - w_t (1 + \tau_t) + (1 - x)\mathbb{E}_t \tilde{\Lambda}_{t+1}^{c} \left[ \frac{\mu(\theta_{t+1}) + \psi'(v_{t+1})}{\mu(\theta_{t+1})} \right] (1 + \tau_{t+1})
\]

where \( \tau_t = \mu_{b,t}/(1 - \mu_{b,t}) \) and \( F_{n,t} \) is the marginal product of labor at period \( t \). Condition
(11) equates the marginal cost of hiring an employee with its marginal benefit net of wages.
Note that in the absence of financial frictions the borrowing constraint disappears and so
does the multiplier \( \mu_b \). The job creation equation in a standard DSGE search model is a
particular case of (11), in which households and capitalists have the same stochastic discount

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\(^5\)Equation (10) can be also written as \( \tilde{J}_{n,t} = \left[ F_{n,t} - w_t \left( 1 + \frac{\mu_{b,t}}{1 - \mu_{b,t}} \right) \right] + (1 - x)\mathbb{E}_t \tilde{\Lambda}_{t+1}^{c} \tilde{J}_{n,t+1} \) where
\( \tilde{J}_n = J_n/(1 - \mu_b) \) and \( \tilde{\Lambda}_{t+1}^{c} \) is equal to the ‘effective’ discount factor of capitalists, \( \Lambda_{t+1}^{c} \left( \mu_{c,t+1}/\mu_{c,t} \right) \).
factor \( (\Lambda_{t+1|t}^t = \Lambda_{t+1|t}^c) \) and there are no credit frictions \( (\mu_b = 0) \):

\[
\frac{\iota \mu(\theta_t) + \psi'(v_t)}{\mu(\theta_t)} = F_{n,t} - w_t + (1 - x) \mathbb{E}_t \Lambda_{t+1|t} \left[ \frac{\iota \mu(\theta_{t+1}) + \psi'(v_{t+1})}{\mu(\theta_{t+1})} \right].
\]

(12)

A comparison of (11) and (12) reveals how credit constraints affect the firm’s ability to create jobs and how credit shocks can have a persistent effect on unemployment. Consistent with other studies that focus on other forms of financial imperfections, financial frictions represented by borrowing constraints create a wedge in the standard job creation equation (Petrosky-Nadeau (2014)). To the extent the constraint is binding, if firms need to finance the costs associated with vacancy posting and with incorporating a worker into the production process, the marginal cost of hiring an employee is increased by a factor of \( \tau_t \). Since the latter is an increasing function of how tightly credit constraints bind, i.e. \( \tau_{\mu_b} > 0 \), a shock that makes the constraint tighter (increases \( \mu_b \)) reduces the ability of firms to create jobs by increasing the marginal cost associated with hiring a worker. Financial constraints introduce a direct mechanism that captures the interaction between credit availability and job creation. Furthermore, now changes in wages affect the marginal value of having an extra worker employed by a factor greater than one \( (1 + \tau) \). Hence, a worsening of credit conditions, i.e. higher \( \mu_b \), increases the sensitivity of the firm’s surplus from a match with respect to changes in wages.

In contrast with Petrosky-Nadeau (2014) and Petrosky-Nadeau and Wasmer (2013), productivity shocks will not be the main source of fluctuations in labor market variables. Under borrowing constraints, productivity shocks besides not generating important movements in the price of capital (the asset used as collateral), move the incentive of firms to post vacancies in the same direction as they move the constraint. Put differently, periods characterized by low realizations of TFP are periods in which the firms are less eager to increase their hiring. Fluctuations in collateral requirements will be the main driving force behind movements in the multiplier \( \mu_b \), and consequently in \( \tau \).

2.7 Wage Bargaining and Equilibrium

Wages are determined as the solution to a generalized Nash bargaining problem between workers and firms. Moreover, I assume that the cost associated with incorporating a newly hired worker into production is paid before negotiation takes place. This implies, as is standard in general equilibrium models with imperfect labor markets, that negotiated wages are imposed to both newly hired and existing workers. Letting \( \phi \in (0, 1) \) be the bargaining power of workers in the negotiation of wages, the wage is a solution to the following problem
\[ w^*_t = \arg\max_{w_t} \mathbb{H}^{1-\phi} \mathbb{H}_m \mathbb{H}^\phi \]  

(13)

where \( \mathbb{H}_{m,t} \) is the household’s marginal value of having one more worker employed. In Appendix A.1, I show that the first-order condition to this problem can be written as,

\[ \varsigma_t \mathbb{J}_{n,t} = (1 - \varsigma_t) \mathbb{H}_{m,t} \]  

(14)

where \( \varsigma_t = \frac{\phi}{\phi + (1 - \phi)(1 + \tau_t)} \). The first-order condition given by equation (14) differs from the usual applications of Nash bargaining in that the firm’s bargaining position, \( \varsigma_t \), is now a function of the credit conditions. With financial frictions, the “effective” marginal product of labor is influenced by the tightness of the credit market. Periods of high collateral requirements are periods in which firms do not benefit significantly from having an extra worker and this strengthens their bargaining position. This contrasts with Monacelli et al. (2011), who find that periods in which credit is tighter are favorable to workers.

The solution to the problem stated in (13), and derived in detail in Appendix A.3, is given by

\[ w^*_t = \phi \left[ F_{n,t}(1 - \mu_{b,t}) + (1 - x)E_t \Lambda^c_{t+1|t} \frac{\mu(\theta_{t+1}) + \psi'(v_{t+1})}{\mu(\theta_{t+1})} \right] + (1 - \phi) \left[ \frac{\phi}{w'(c_t)} + s \right] \]

\[ - \phi(1 - x)E_t \Lambda^h_{t+1|t} \left\{ [1 - f(\theta_{t+1})] \frac{\mu(\theta_{t+1}) + \psi'(v_{t+1})}{\mu(\theta_{t+1})} \right\} \]  

(15)

Note that, in the absence of financial frictions and with \( \Lambda_{t+1|t}^c = \Lambda_{t+1|t}^h \), the wage would collapse to a static split, which is the solution to standard labor-search model with frictionless financial markets.

Appendix A.2 presents the definition of the recursive equilibrium. Having discussed all the aspects of the model economy, the next section quantitatively assesses the model.

3 Quantitative Analysis

3.1 Benchmark Parametrization

The functional forms for preferences, adjustment costs, and vacancy costs are presented in Table 1. The functional form assumed for the capital adjustment costs is commonly used in the literature. As is standard, the parameters \( \omega_1 \) and \( \omega_2 \) are chosen so that in the steady state \( \Phi_i(i/k) = 1 \) and \( i = \delta k \). This implies that \( \omega_1 = \delta \xi \) and \( \omega_2 = -\xi \delta / (1 - \xi) \). As in Perri and Quadrini (2011), capitalists are assumed to have a standard CRRA utility function. As
is common in the literature, the function $\psi(v_t)$ is linear in vacancies.

### Table 1: Functional Forms

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
<th>Functional form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u(c_t)$</td>
<td>Utility function of households</td>
<td>$\ln(c_t)$</td>
</tr>
<tr>
<td>$u(d_t)$</td>
<td>Utility function of capitalists</td>
<td>$d_1^{1-\sigma} - \sigma t - 1$</td>
</tr>
<tr>
<td>$\Phi(i_t/k_t)$</td>
<td>Capital adjustment cost</td>
<td>$\frac{\omega_1}{1-\xi} \left( \frac{i_t}{k_t} \right)^{1-\xi} + \omega_2$</td>
</tr>
<tr>
<td>$\psi(v_t)$</td>
<td>Vacancy creation cost</td>
<td>$\kappa v_t^\chi$</td>
</tr>
</tbody>
</table>

The baseline values for parameters can be seen in Tables 2 and 3. The set of parameters that are determined using targets, presented in the first panel of Table 3, is given by $\{\beta^h, \beta^c, \varphi, \xi, \nu, \eta, \sigma_z, \sigma_y\}$. The unit of time is taken to be a quarter. All the data used in this section are described in detail in Appendix E.

The discount factors for households, $\beta^h$, and capitalists, $\beta^c$, are set to target, respectively, an annual steady-state interest rate of 1.6% and an annual steady-state return on equity of 7%. As is standard in the literature, the elasticity of capital in the production function, $\alpha$, is set to 0.34 while the quarterly depreciation rate for capital, $\delta$, is set to 2.5%. The parameter that regulates the sensitivity of the capital adjustment cost function, $\xi$, is set to match the empirical volatility of the detrended physical capital stock.

The exogenous separation rate, $x$, is set to 10% which corresponds to a middle value between the ones chosen by Andolfatto (1996) and Merz (1995), and it is also the value used by Shimer (2005). Start-up or training costs, $\iota$, are set to 0.9, implying that in steady state, training costs represent slightly less than 150% of monthly wages. This value is within the range of estimates of training costs for U.S. firms found in the literature. The cost of posting vacancies, $\kappa$, imply that in steady state vacancy costs represent 0.65% of output ($\bar{v}\kappa/\bar{y} = 6.5\%$) a value that is also within the range found in other studies. The elasticity of matches with respect unemployment is taken from Chugh (2013) and the worker’s bargaining power is set to 0.4 which is in the middle range of the values found in the literature.

The parameter that governs the disutility of labor, $\varphi$, unemployment benefits, $s$, and the efficiency of the match, $\nu$, are chosen to simultaneously match a steady-state unemployment rate of 10%, a steady-state replacement ratio, $s/\bar{w}$, of 0.22, and quarterly job filling rate, $\mu(\theta)$, of 0.9. Since the model does not account for non-participation, the target for unemployment is higher than the average U.S. unemployment rate so that unemployment in the model can be interpreted as being a mix between unemployed and out of the labor force workers. As

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6For instance, Barron et al. (1997) find that for the period between 1980 and 1993 training costs accounted for 34 to 156% percent of monthly wages.
a higher replacement ratio would imply, *ceteris paribus*, a higher volatility of labor market variables, the target for that moment is chosen to be, again, in the lower spectrum of values in the literature.

Table 2: Value of parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_h$</td>
<td>Household’s discount factor</td>
<td>0.996</td>
</tr>
<tr>
<td>$\beta_c$</td>
<td>Capitalists’ discount factor</td>
<td>0.983</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Disutility of labor</td>
<td>0.541</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Investment adjustment cost</td>
<td>0.050</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Efficiency of the match</td>
<td>0.651</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Share of capital in the production function</td>
<td>0.34</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Agents’ relative risk-aversion</td>
<td>2</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Separation rate</td>
<td>0.10</td>
</tr>
<tr>
<td>$s$</td>
<td>Unemployment benefits</td>
<td>0.40</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Elasticity of matches w.r.t. unemployment</td>
<td>0.50</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Worker’s bargaining power</td>
<td>0.40</td>
</tr>
<tr>
<td>$\iota$</td>
<td>‘Start-up’ or ‘Training’ cost</td>
<td>0.90</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Vacancy cost</td>
<td>0.18</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Curvature of vacancy creation costs</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3: Parameters for Stochastic Processes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\eta}$</td>
<td>Steady-state credit market tightness</td>
<td>0.3086</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Persistence of aggregate productivity</td>
<td>0.9457</td>
</tr>
<tr>
<td>$\rho_\eta$</td>
<td>Persistence of credit shock</td>
<td>0.9703</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>Standard deviation of productivity shock</td>
<td>0.0082</td>
</tr>
<tr>
<td>$\sigma_\eta$</td>
<td>Standard deviation of credit shock</td>
<td>0.0132</td>
</tr>
</tbody>
</table>

The standard deviations for the productivity shock, $\sigma_z$, and the credit shock, $\sigma_\eta$, are jointly calibrated so the model’s standard deviations for both output, $\sigma_y$, and debt-to-output ratio, $\sigma_y/b$, match their empirical counterparts. The debt-to-output ratio also disciplines the mean of the process that governs the credit shock, $\bar{\eta}$. Setting the steady-state collateral requirement to 0.3086 (30.68% of the value of physical capital stock) delivers a debt-to-output ratio equal to 1.763 which is the value found in the data for the period being considered. The parameters that govern the persistence of both shocks are taken from the estimates of Jermann and Quadrini (2012).

Finally, given that the model contains a large number of continuous state variables, I implement the second-order perturbation method as described by Schmitt-Grohe and Uribe.
The next sections present and discuss the numerical results. I start by presenting results from impulse responses of one standard deviation shocks to aggregate productivity and to collateral requirements that occur in period 1. In the following graphs the scale represents percentage deviations (log-deviations) from the steady state, except for the job finding rate, which represents percentage point deviations.

3.1.1 Impulse Response: TFP Shock

Figure 1 shows the response of some of the variables of the model to a one standard deviation positive shock to aggregate productivity. Most of these movements conform with standard intuition. Following a positive technology shock firms increase their hiring, with vacancies increasing on impact almost by 2%. At the same time, unemployment decreases, causing an even larger increase in labor market tightness, $\theta$, and a movement along the Beveridge curve. The higher marginal product of capital, as well as its higher collateral value, explain the positive response of investment. Note that the hump-shaped dynamics of the latter are due to the presence of adjustment costs.

The persistence of the shock translates into persistence in the variables of interest as well. The increase in household’s consumption is strong. As capitalists increase their borrowing in response to the productivity shock, the interest accrued on higher debt holdings together with the rise in wages, allows households to afford a higher level of consumption.

The propagation generated by the model can be observed in Figure 2. The figure shows the impulse response of some of the variables of interest in the model without financial frictions, as well as in the baseline case, to a productivity shock. Specifically, for the former, I solve the baseline model with two main modifications: i) there are no financial frictions and ii), households and capitalist discount the future at the same rate. Furthermore, as in Merz (1995) and Andolfatto (1996), firms in this environment do not have access to an inter-period loan. The model without financial frictions is calibrated such that the model has the same non-stochastic steady state as the baseline model. In addition, the standard deviation of productivity shock is set to match the empirical volatility of detrended output. A legitimate concern is whether the propagation generated by the model is due to a higher share of fixed costs in the firm’s total hiring costs. In order to address this, Figure 2 also shows, in a dotted line, the impulse response of the model without financial frictions but in which the structure of hiring costs is such that fixed costs have the same relative importance as in the baseline model (i.e., the model with frictionless financial markets has the same ratio of fixed to total costs as in the baseline model). Clearly, the sluggish behavior of labor market variables is not being driven by the relative importance of fixed costs.
The response from the model without credit constraints is the DSGE equivalent of the responses of a standard MP model and they are in line with the findings of Fujita and Ramey (2007): a standard search model cannot reproduce the sluggish response of labor market variables to productivity shocks. Fujita and Ramey (2007) show that the \( v/u \) ratio continues to increase for nearly 4 quarters after an increase in productivity has occurred. In the MP model, as well as in the model without financial frictions, the adjustment is instantaneous and so the model generates almost no propagation of productivity shocks in the labor market variables. Credit constraints improve the performance of a standard DSGE model by quantitatively accounting for two empirical features that those models are not able to replicate.\(^7\) First, both in the data and in the model nearly 60% of the total increase in labor market tightness occur in the periods that follow the exogenous increase in productivity (60% in the data vs. 62.4% in the model). Second, the \( v/u \) ratio continues to increase for several periods after the shock. While in the data the increase in labor market tightness that follows a positive productivity shock propagates for four quarters, the model generates a \( v/u \) ratio that increases for three consecutive quarters. The solely presence of credit frictions brings the dynamics of model closer to the data.

The mechanism is simple. In the model with financial frictions, a positive productivity shock generates a tightening of the credit constraint. This makes capital relatively more valuable due to its use as collateral. Therefore, firms would increase investment by more on impact augmenting, in this way, their physical capital stock in order to relax the constraint

\(^7\)See Fujita and Ramey (2007) and Epstein (2012).
and ‘take advantage’ of the higher productivity. This preference of firms for building the capital stock first is what generates a delayed response of vacancies and the consequent sluggish response of the $v/u$ ratio. While credit constraints improve the ability of a standard DSGE-search model in propagating productivity shocks, I show that a framework with only those disturbances is not able to generate important movements in the extensive margin.

![Graphs showing the effects of credit constraints on various economic indicators.](image)

**Figure 2: Effects of Credit Constraints**

### 3.1.2 Impulse Response: Credit Shock

Figure 3 plots the response to a one standard deviation negative shock to credit market tightness during period 1. In other words, firms’ leverage is exogenously decreased and firms are able to borrow a smaller fraction of their collateral. Since credit shocks do not generate persistent dynamics, the plots only show the response for 20 periods.

The credit shock decreases the firm’s ability to access funding and the firm responds by decreasing investment, job creation, and borrowing. The reduction of investment decreases future levels of the capital stock, which in turn further decreases the firm’s ability to borrow, thus serving as an endogenous propagation mechanism. The persistent reduction in investment follows from the credit constraint. Since credit is tighter for several periods, the value of relaxing the constraint is higher and long-lasting (the increase in $\mu_b$ is persistent), which in turn causes a prolonged increase in the marginal cost of investing.

With respect to labor market variables, even though the behavior of vacancies is not very persistent, it is important on impact. The magnitude of the immediate response of vacancies leads to a persistent increase in unemployment. The full recovery of employment
takes almost 1.5 years. These impulse responses also show that the behavior of wages after an increase in collateral requirements is in contrast with Monacelli et al. (2011) whose baseline framework generates increasing wages during contractions caused by a credit tightening. In the present paper, following a tightening of credit conditions, the model with competitive firms is able to generate a rise in the unemployment rate despite declining wages. This allows the model to generate, consistent with the data, procyclical wages.\(^8\)

\[\begin{align*}
\text{Output} & \quad 0.4 \\
\text{Investment} & \quad 1 \\
\text{Job-finding rate} & \quad 1 \\
\text{Unemployment rate} & \quad 6 \\
\text{Vacancies} & \quad 5 \\
\text{Wage} & \quad 0.1 \\
\text{Dividends} & \quad 0.4 \\
\text{Consumption} & \quad 0.3 \\
\text{Bonds} & \quad 0.1 \\
\end{align*}\]

Figure 3: Negative Credit Shock

It is important to mention that, while the framework with credit shocks and financial frictions improve the performance of the model in several dimensions related to labor markets, it suffers from a shortcoming common to models with financial shocks. As is well known in closed economy models with these type of disturbances, there is a counterfactual increase in consumption following a borrowing tightening. Following the negative shock, firms reduce their level of borrowing. This deleveraging is consistent with recent empirical evidence that suggests periods in which the ability to borrow is reduced are characterized by a reduction in the firm’s debt holdings. This, in turn, implies a reduction in the payment of interest which allows firms to distribute more dividends (although the increase in dividends payout is very minor). In other words, the negative shock reduces the firm’s ability to borrow, which in turn diminishes loan liabilities and therefore leads to an increase in net worth. The household, responds on impact by increasing their consumption since the reduction in interest rates decreases the incentives to save. In later periods, however, both lower wages

\(^8\)The unconditional correlation of wages and output in the model is 0.91 while in the data, depending on what measure of worker’s compensation is used, the correlation is in the range 0.32–0.91.
and the reduced interest payments received from firms bring consumption below the steady state and allows the model to generate a procyclical consumption.

### 3.2 Business Cycles Statistics

Table 4 shows some standard business cycle statistics from the data alongside their counterparts from the model’s simulation. Several changes in business cycles have occurred since the mid-eighties and because accounting for them is not the goal of this paper, the data used spans first quarter of 1984 to the last quarter of 2011. Each moment is calculated from the difference between the log level of each series and the trend obtained by filtering the data using a Hodrick-Prescott filter with a smoothing parameter of 100,000 as in Shimer (2005). From the business cycle statistics it can be seen that the model performs quantitatively well in accounting for most of the cross-correlations and volatilities for the U.S. economy for the post-1984 period.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>0.0202</td>
<td>0.0202</td>
</tr>
<tr>
<td>$y$</td>
<td>1.1668</td>
<td>0.0762</td>
</tr>
<tr>
<td>$u$</td>
<td>0.1599</td>
<td>0.1216</td>
</tr>
<tr>
<td>$v$</td>
<td>0.3137</td>
<td>0.1277</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.0836</td>
<td>0.0559</td>
</tr>
<tr>
<td>$i$</td>
<td>0.0106</td>
<td>0.0106</td>
</tr>
</tbody>
</table>

Table 4: Unconditional Business Cycles Statistics

The model accounts for roughly 40% of the variations observed in labor market tightness and more than 75% of the variation in vacancies. However, it falls short in generating fluctuations in unemployment of similar magnitude to those in the data. Nevertheless, unemployment volatility is almost four times (3.8 times) that of output, which as seen in Table 9 By using this value, the transformed series will exhibit cyclical fluctuations both at the short as well as at the medium frequency and, loosely speaking, the model will have ‘more fluctuations to match’. Another recall that the volatility of output was a target of the calibration so it is not a surprise that the model matches its value. The same is true with respect to the capital stock.
7, is a significant improvement from a labor search model without financial frictions and fluctuations in credit conditions. Introducing financial frictions that arise from a costly state verification problem, Petrosky-Nadeau (2014) presents a model that generates an unemployment rate that is 2.37 times more volatile than output. In this paper, firms have the possibility of accumulating physical capital as well as using a one-period financial instrument, both features absent in Petrosky-Nadeau (2014). The model presented here does a better job generating fluctuations in unemployment, even though both debt and capital are additional channels by which firms can respond to shocks. This framework also captures the Beveridge curve – the strong negative relationship between vacancies and unemployment – observed in the data.

3.3 Importance of Credit Shocks

To evaluate the contribution of credit shocks in generating fluctuations in labor market variables, I simulate the model with one shock at a time. Table 5 reports the results obtained from simulating the model with only aggregate productivity shocks and another simulation with only credit disturbances.

Relative to the benchmark, the model with only productivity shocks does a better job in matching the cross correlations of labor market variables and output observed in the data. Nevertheless, it is clear that the model lacks amplification in terms of labor market variables. In particular, fluctuations in job creation and labor market tightness are reduced by more than 70% and 60%, respectively, relative to the model in which both shocks are present.

The significance of credit shocks in generating cyclical movements in labor market variables is made clear by looking at the behavior of the model with only those disturbances. The relative importance of credit shocks can be traced back to the job creation equation. Relative to a standard model, the wedge introduced in the job creation equation in the model with borrowing constraints acts as a mechanism that amplifies the dynamic response of firms to shocks that reduce their ability to borrow. Since that wedge is an increasing function of the degree of tightness in the credit market, it is useful to study the dynamics of the multiplier on the credit constraint, $\mu_b$, following a negative shock. Figure 4a reports the impulse response for both a negative credit and TFP shock.

By directly affecting the borrowing constraint, financial shocks have a significantly greater impact in the degree of credit tightness faced by firms. In addition, as previously discussed, an aggregate productivity shock moves the constraint and incentives to both invest and post vacancies in the same direction. Hence, it is not surprising that following a negative TFP shock the credit constraint becomes less tight. Captured by equation (10), the increase in
credit tightness that follows a negative (positive) credit (TFP) shock reduces the net benefit than a new worker brings to the firm, reducing the incentives of the firm to create jobs. As the degree of credit tightness experienced by the firm is relatively more sensitive to changes in credit conditions, credit shocks are able to generate a greater response of job creation and, consequently, of unemployment. Analogous reasons explain the responses in Figure 4b that shows the dynamics of the effective bargaining position of workers, $\varsigma$. Almost half of the variations in unemployment, vacancies, and labor market tightness can be accounted for by only having fluctuations in collateral requirements.

Table 5: Business Cycles Statistics

<table>
<thead>
<tr>
<th></th>
<th>Model with only TFP shocks</th>
<th></th>
<th>Model with only credit shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>0.0195</td>
<td>0.0051</td>
<td>0.0656</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.9652</td>
<td>0.9707</td>
<td>0.6507</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.9646</td>
<td>0.8182</td>
<td>0.6078</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.9835</td>
<td>0.9453</td>
<td>0.5282</td>
</tr>
<tr>
<td>$i$</td>
<td>0.9451</td>
<td>-0.8829</td>
<td>-0.5953</td>
</tr>
<tr>
<td>$k$</td>
<td>0.9495</td>
<td>-0.8829</td>
<td>-0.5953</td>
</tr>
<tr>
<td>Correlation $v$</td>
<td>0.9698</td>
<td>0.9649</td>
<td>0.9618</td>
</tr>
<tr>
<td>Correlation $\theta$</td>
<td>0.9811</td>
<td>1.0024</td>
<td>0.9424</td>
</tr>
<tr>
<td>Correlation $i$</td>
<td>0.9618</td>
<td>1.0024</td>
<td>0.9424</td>
</tr>
<tr>
<td>Correlation $k$</td>
<td>0.9618</td>
<td>1.0024</td>
<td>0.9424</td>
</tr>
</tbody>
</table>

While there are caveats when interpreting variance decompositions with a small number of shocks, Table 6 shows the relative importance of each shock in accounting for the fluctuations in some of the variables of the model. Each value in the table corresponds to the percentage of the total variation of a variable (rows) that is explained by each shock (columns). The high responsiveness of wages to productivity shocks is reflected in a high elasticity of wages with respect to productivity, which is equal to 0.8598.\(^{11}\) Given this high elasticity, it is not surprising that the model with only TFP shocks does not generate important movements in labor market quantities. On the other hand, the elasticity of wages with respect to credit shocks equal to 0.2744, so that wages are relatively quite unresponsive with respect

\(^{11}\)This elasticity, which can be denoted by $\zeta_{z,w}$, is calculated as the product of the correlation coefficient between wages and productivity (0.9401), and the relative standard deviations of wages and productivity (0.9146); formally $\zeta_{z,w} = \rho_{z,w} (\sigma_w/\sigma_z)$, which is the regression coefficient of wages on productivity, with both variables in log-scale.
to financial disturbances.

Figure 4: Effects of Negative Shocks on Tightening and Bargaining

Table 6: Variance Decomposition

<table>
<thead>
<tr>
<th>TFP Shock</th>
<th>Credit Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>93.64</td>
</tr>
<tr>
<td>$u$</td>
<td>25.99</td>
</tr>
<tr>
<td>$v$</td>
<td>6.99</td>
</tr>
<tr>
<td>$\theta$</td>
<td>14.07</td>
</tr>
<tr>
<td>$w$</td>
<td>94.19</td>
</tr>
</tbody>
</table>

To emphasize the contribution of both credit shocks and financial constraints, Table 7 compares the results obtained from the model’s simulations. In all the specifications, the size of the productivity shock was chosen to match the volatility of output in the data so the standard deviation of output is the same across the columns.

Table 7: Standard Deviations: Data, Financial Frictions, and Credit Shocks

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>0.1668</td>
<td>0.0330</td>
<td>0.0403</td>
<td>0.0762</td>
</tr>
<tr>
<td>$v$</td>
<td>0.1599</td>
<td>0.0319</td>
<td>0.0333</td>
<td>0.1216</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.3137</td>
<td>0.0431</td>
<td>0.0497</td>
<td>0.1277</td>
</tr>
<tr>
<td>$w$</td>
<td>0.0155</td>
<td>0.0169</td>
<td>0.0150</td>
<td>0.0150</td>
</tr>
<tr>
<td>$b/y$</td>
<td>0.0498</td>
<td>–</td>
<td>0.0258</td>
<td>0.0498</td>
</tr>
</tbody>
</table>

As shown before, the presence of credit constraint generate a more sluggish response of labor market variables with respect to productivity shocks. However, financial constraints,
per se, improve only slightly the performance of a standard general equilibrium search model in generating fluctuations in labor market quantities. It is the combination of financial constraints and exogenous fluctuations in collateral requirements that significantly help to close the gap between the model and the data.

4 Conclusion

This paper examines how firms that are constrained in their ability to borrow are affected by fluctuations in financial conditions and, in particular, how these constraints affect both their capacity and incentives to post vacancies and create new jobs.

I find that an environment in which credit constraints that are based on the physical capital stock owned by the firm can quantitatively reproduce the empirical sluggish response of labor market variables that follows productivity shocks. Relative to standard DSGE search models, financial constraints introduce a wedge in the job creation equation so that the relative bargaining position of firms is influenced by credit conditions. In the model, periods of tighter credit are periods that are more favorable to the firm.

I show that fluctuations in collateral requirements generate significant movements in labor market variables. While productivity shocks are important for generating fluctuations in aggregates such as output and investment, credit shocks have significant effects on variables such as unemployment, vacancy posting, and labor market tightness. Because changes in collateral requirements do not entirely translate into changes in wages, these disturbances have a large impact on the ability of firms to create jobs. Contrary to the effects of productivity shocks, the adjustment that follows from changes in credit conditions is mainly through quantities and not prices. Fluctuations in collateral requirements are, hence, promising in explaining business cycle movements in labor market variables.
References


A Derivations

A.1 Nash Bargaining

In order to set up the optimization problem, I need to define the marginal value of having a family member matched from the household perspective, $\mathbb{H}_{m,t}$. The fraction of household members that are employed at period $t$ evolves according to

$$n_{h,t} = (1 - x)n_{h,t-1} + f(\theta_t)u_t.$$

Let $\mathbb{H}_{n,t}$ be the value function associated with having an extra member employed; since separations occur at the end of the period, this value is given by

$$\mathbb{H}_{n,t} = -\phi + \mu_{h,t}w_t + \beta_h \mathbb{E}_t \{ x[1 - f(\theta_{t+1})] \mathbb{H}_{n,t+1} + [1 - x + xf(\theta_{t+1})] \mathbb{H}_{n,t+1} \} \quad (A.1)$$

where $\mu_{h,t}$ is the Lagrange multiplier on the budget constraint and $\mathbb{H}_{u,t}$ is the value of having an extra family member unemployed next period. Specifically, the value of having an extra unemployed member at time $t$ is given by

$$\mathbb{H}_{u,t} = \mu_{h,t}s + \beta_h \mathbb{E}_t \{ f(\theta_{t+1})\mathbb{H}_{n,t+1} + [1 - f(\theta_{t+1})] \mathbb{H}_{u,t+1} \}. \quad (A.2)$$

The household’s marginal surplus of a match is therefore defined as

$$\mathbb{H}_{m,t} = \frac{\mathbb{H}_{n,t} - \mathbb{H}_{u,t}}{\mu_{h,t}}$$

which after some algebra can be shown to be,

$$\mathbb{H}_{m,t} = \frac{\phi}{\mu_{h,t}} + w_t - s + (1 - x)\mathbb{E}_t \Lambda_{t+1|t}^h [1 - f(\theta_{t+1})] \mathbb{H}_{m,t+1} \quad (A.3)$$

where $\Lambda_{t+1|t}^h$ is the household’s discount factor, equal to $\beta_h \mathbb{E}_t \frac{\mu_{h,t+1}}{\mu_{h,t}}$.

Using the fact that in equilibrium $n_{c,t} = n_{h,t} = n_t$, the marginal value of a worker to the firm, $\mathbb{J}_{n,t}$, can be expressed as

$$\mathbb{J}_{n,t} = (1 - \alpha)z_t \left( \frac{k_t}{n_t} \right)^{\alpha} - w_t(1 + \tau_t) + \mathbb{E}_t \tilde{\Lambda}_{t+1|t}^c (1 - x)\mathbb{J}_{n,t+1} \quad (A.4)$$

where $\tau_t = \mu_{b,t}/(1 - \mu_{b,t})$, and $\tilde{\Lambda}_{t+1|t}^c$ is the capitalist’s modified stochastic discount factor which is equal to $\mathbb{E}_t \Lambda_{t+1|t}^c \frac{\mu_{c,t+1}}{\mu_{c,t}}$. 

\[12\]State variables are being omitted for the sake of notation.
Now that both the value functions for the household and the entrepreneur are defined, I can formalize the problem explicitly. The wage that solves the generalized Nash Bargaining maximizes, every period, the weighted geometric average of the gains from trade,

\[ w^*_t = \arg\max_{w_t} \sum_{n,t}^{1-\phi} H_{m,t}^\phi \]

where \( \phi \in (0,1) \) is the bargaining power of workers in the process of wage negotiation. The solution to this problem is given by the standard Nash bargaining rule,

\[ \zeta_t \sum_{n,t} = (1 - \zeta_t) H_{m,t} \]

where \( \zeta_t = \frac{\phi}{\phi + (1-\phi)(1+\tau_t)}. \)\(^{13}\)

After rearranging some terms the joint surplus of the match, \( S_t \) can be written as

\[ S_t = \sum_{n,t} + H_{m,t} = F_{n,t} - w_t - \left( \frac{\phi}{u'(c_t)} + s \right) + (1-x)E_t \left\{ \hat{\Lambda}_{t+1|t}\sum_{n,t+1} + \Lambda_{t+1|t}^h [1 - f(\theta_{t+1})] H_{m,t+1} \right\} \]

where \( F_{n,t} \) is the marginal product of labor at time \( t \) and I am using the optimality condition that \( \mu_{h,t} = u'(c_t) \). Using equation (A.6) and the fact that \( S_t = \sum_{n,t} + H_{m,t} \), I can write \( \sum_{n,t} = (1 - \zeta_t) S_t \) and \( H_{m,t} = \zeta_t S_t \). Using the both the former and the latter in the definition of the joint surplus,

\[ S_t = F_n - \tau_t w_t - s - \left( \frac{\phi}{u'(c_t)} \right) + (1-x)E_t \left\{ \hat{\Lambda}_{t+1|t}\sum_{n,t+1} + \Lambda_{t+1|t}^h [1 - f(\theta_{t+1})] \frac{S_{t+1}}{1 - \zeta_{t+1}} \sum_{n,t+1} \right\}. \]

Multiplying both sides of equation (A.7) by \( (1 - \zeta_t) \),

\[ (1 - \zeta_t) S_t = (1 - \zeta_t) \left( F_n - \tau_t w_t - s - \left( \frac{\phi}{u'(c_t)} \right) \right) + (1 - \zeta_t)(1-x)E_t \left\{ \hat{\Lambda}_{t+1|t}\sum_{n,t+1} + \Lambda_{t+1|t}^h [1 - f(\theta_{t+1})] \frac{S_{t+1}}{1 - \zeta_{t+1}} \sum_{n,t+1} \right\}. \]

Finally, substituting in (A.8) the job creation condition, \( \sum_{n,t} = (1 - \zeta_t) S_t = \mu(\theta_t) + \psi'(v_t) \mu(\theta_t) (1 + \tau_t) \), the definition of \( \zeta_t \), and solving for \( w_t \) and arranging terms, the wage that solves the Nash

---

\(^{13}\) The first order necessary condition of equation (A.5) is given by \( \phi \frac{\partial H_{m,t}}{\partial w_t} \sum_{n,t} + (1 - \phi) \frac{\partial \sum_{n,t}}{\partial w_t} H_{m,t} = 0. \)
bargaining is obtained:

\[
w_t^* = \frac{\phi}{1 + \tau} \left[ F_n + (1 - x) E_t \tilde{\Lambda}_{t+1}^c \left( \frac{\mu(\theta_{t+1}) + \psi'(v_{t+1})}{\mu(\theta_{t+1})} (1 + \tau_{t+1}) \right) \right] + (1 - \phi) \left[ \frac{\varphi}{w'(c_t)} + s \right]
\]

\[-\phi(1 - x) E_t \Lambda_{t+1}^h \left\{ (1 - f(\theta_{t+1})) \frac{\mu(\theta_{t+1}) + \psi'(v_{t+1})}{\mu(\theta_{t+1})} \right\}.
\]

After taking into account the definitions of \( \tilde{\Lambda}_{t+1}^c \) and \( \tau_t \), (A.1) can be expressed as,

\[
w_t^* = \phi F_n + (1 - \phi) \left[ \frac{\varphi}{w'(c_t)} + s \right] + \phi(1 - x) E_t \Lambda_{t+1}^c \left[ \mu(\theta_{t+1}) + \psi'(v_{t+1}) \right] \theta_{t+1}
\]

which corresponds to equation (15) in the main text. As mentioned in the main text, in the absence of financial frictions and with \( \Lambda_{t+1}^c = \Lambda_{t+1}^h \), the wage would collapse to a static split:

\[
w_t^* = \phi F_n + (1 - \phi) \left[ \frac{\varphi}{w'(c_t)} + s \right] + \phi(1 - x) E_t \Lambda_{t+1}^c \left[ \mu(\theta_{t+1}) + \psi'(v_{t+1}) \right] \theta_{t+1}
\]

**A.2 Model With Financial Frictions**

Recall that the problem of the firm is given by

\[
\mathbb{J}(\omega^c_t; \Omega_t) = \max_{\{d_t, \nu_t, i_t, k_{t+1}, b_{t+1}\}} \{ d_t + E_t \Lambda_{t+1}^c(\omega^c_{t+1}; \Omega_{t+1}) \}
\]

subject to

\[
z_k n_{c,t}^{1-\alpha} + \frac{b_{t+1}}{R_t} = d_t + b_t + \nu_t n_{c,t} + i_t + \nu_t \mu(\theta_t) + \psi(v_t) \quad (A.10)
\]

\[k_{t+1} = (1 - \delta)k_t + \Phi \left( \frac{i_t}{k_t} \right) k_t \quad (A.11)
\]

\[d_t + \nu_t n_{c,t} + \nu_t \mu(\theta_t) + \psi(v_t) + i_t + b_t \leq \eta q_t k_t \quad (A.12)
\]

\[n_{c,t} = (1 - x)n_{c,t-1} + \nu_t \mu(\theta_t) \quad (A.13)
\]

When optimizing, the individual firm takes as given the probability that a vacancy will be filled, \( \mu(\theta_t) \), the gross interest rate, \( R_t \), the stochastic discount factor of capitalists, \( \Lambda_{t+1}^c \), and the wage, \( w_t \).

Letting \( \mu_{c,t}, \mu_{k,t}, \mu_{b,t}, \) and \( \mu_{e,t} \) denote, respectively, the multipliers on the budget constraint, (A.10), the capital law of motion, (A.11), the borrowing constraint, (A.12), and the
law of motion for employment, \((A.13)\), the first-order conditions for \(d_t, i_t, k_{t+1}, \) and \(v_t\), are

\[
\mu_{c,t} = 1 - \mu_{b,t} \tag{A.14}
\]

\[
\frac{1}{R_t} = \mathbb{E}_t \Lambda_{t+1}^c \left( \frac{1}{1 - \mu_{b,t}} \right) \tag{A.15}
\]

\[
\mu_{k,t} = \frac{1}{\Phi_{i_t}} \tag{A.16}
\]

\[
\mu_{k,t} = \mathbb{E}_t \Lambda_{t+2}^c \left\{ F_{k,t+1} (1 - \mu_{b,t+1}) - \frac{i_t}{k_t} + \mu_{k,t+1} \left[ 1 - \delta + \Phi \left( \frac{i_t}{k_t} \right) \right] \right\} + \mathbb{E}_t \Lambda_{t+1}^c \mu_{b,t+1} \eta_{t+1} q_{k,t+1} \tag{A.17}
\]

\[
\mu_{e,t} = \iota \mu(\theta_t) + \psi'(v_t) \tag{A.18}
\]

where \(q_k\) is defined as the ratio of two lagrangian multipliers, \(\mu_k/\mu_c\), and \(\Phi_i\) and \(F_k\) are, respectively, the first derivatives of the investment adjustment cost function with respect to investment and the marginal product of capital.

The job-creation equation is obtained by combining Equation (10) in the main text with the first order condition with respect to vacancies (Equation (A.18)):

\[
\frac{\iota \mu(\theta_t) + \psi'(v_t)}{\mu(\theta_t)} = (1 - \alpha) z_t \left( \frac{k_t}{n_{c,t}} \right)^\alpha (1 - \mu_{b,t}) - w_t + (1 - x) \mathbb{E}_t \Lambda_{t+1}^c \left[ \frac{\iota \mu(\theta_{t+1}) + \psi'(v_{t+1})}{\mu(\theta_{t+1})} \right]. \tag{A.19}
\]

The equilibrium in the economy then defined as follows.

**Definition 1 (Equilibrium)** A recursive equilibrium is defined as a set of i) firm’s policy functions \(d(\omega^c; \Omega), n_{c}(\omega^c; \Omega), k(\omega^c; \Omega), b(\omega^c; \Omega), i(\omega^c; \Omega), \) and \(v(\omega^c; \Omega)\); ii) household’s policy functions \(c(\omega^h; \Omega), n_{h}(\omega^h; \Omega), \) and \(a(\omega^h; \Omega)\); iii) a lump sum tax \(T\), iv) prices \(w(\Omega)\) and \(R(\Omega)\); and v) law of motion for the aggregate states, \(\Omega' = \Psi(\Omega)\), such that: i) firms’ policies satisfy conditions \((A.14)-(A.17)\) and \((A.19)\); ii) household’s policy function satisfies \((5)\); iii) the wage is determined by \((15)\); iv) \(R(\Omega)\) clears the market for the riskless asset such that \(a(\Omega) = b(\Omega)\); v) labor demanded by firms is equal to labor supplied by workers, \(n_{c}(\omega^c; \Omega) = n_{h}(\omega^h; \Omega)\); vi) the law of motion \(\psi(\Omega)\) is consistent with individual decisions and with the stochastic processes for \(z\) and \(\eta\), and vii) the government has a balanced budget so that \(s(1 - n) = T\).

### A.3 Steady-State Borrowing Constraint

Here I show that the credit constraint is binding in the steady state. From the household’s first order conditions with respect to the one-period bond we have that

\[
\frac{1}{R_t} = \beta_h \frac{u'(c_{t+1})}{u'(c_{t+1})}, \tag{A.20}
\]
Similarly, for entrepreneurs
\[
\frac{1}{R_t} = \mathbb{E}_t\Lambda_{t+1}\left( \frac{1}{1 - \mu_{b,t}} \right)
\]  
(A.21)
where \( \mu_b \) is the lagrange multiplier on the credit constraint. Combining (A.20) and (A.21) and denoting as \( \bar{\mu}_b \) the steady state value of \( \mu_b \), we have that in the steady state
\[
\frac{\beta_h - \beta_c}{\beta_c} = \bar{\mu}_b.
\]  
(A.22)
Therefore, as long as households are more patient than entrepreneurs, i.e. \( \beta_h > \beta_c \), the borrowing constraint is binding in steady state (\( \bar{\mu}_b > 0 \)).

\section*{B Robustness Exercises}

This section examines the sensitivity of the results to changes in different parameters of the model and it performs some quantitative experiments.

\subsection*{B.1 Investment Adjustment Costs}

An important parameter of the model is the curvature of the investment adjustment cost function, which directly affects the value of capital, \( q_k \). Therefore, it is important to examine whether the performance of the model in the baseline case is driven by the relatively high value for the parameter that regulates the magnitude of those costs, \( \xi \). In this exercise I recalibrate the model to match the same non-stochastic steady state of the baseline case. The same is true for the productivity and credit shock, i.e. under both cases (low and high capital adjustment costs), the model matches the empirical standard deviations of both output and debt-to-output ratio.\(^{14}\) Table B1 presents those results.

The conclusion regarding the relative importance of fluctuations in credit conditions in generating movements in labor market variables, holds under these different parameterizations. Reducing the value of \( \xi \) from 0.055 to 0.01 (slightly over an 80\% reduction) does not significantly affect the results obtained in previous sections. Facing lower marginal costs of adjusting the capital stock, firms are more willing to respond to shocks by adjusting on that dimension. This, as expected, increases the volatility of investment. By decreasing the costs of adjusting investment the costs of adjusting employment are relatively higher, leading to a small reduction in the volatility of labor market variables. Similarly, the results remain

\(^{14}\)Even though the adjustment costs in the lower panel are labeled as ‘high’, they are in the lower spectrum of the values found in the literature (for instance, Perri and Quadrini (2011) use a value of 0.5).
robust to after increasing slightly over 80% the costs associated with adjusting the capital stock (from 0.055 to 0.1). As expected, the volatility of labor market variables is now higher.

Table B1: Importance of Investment Adjustment Costs

<table>
<thead>
<tr>
<th>Small Capital Adjustment Costs ($\xi = 0.01$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
</tr>
<tr>
<td>$y$</td>
</tr>
<tr>
<td>$u$</td>
</tr>
</tbody>
</table>

Correlation Matrix

| $v$      | 1 | 0.9569 | 0.6643 | 0.0802 |
| $\theta$ | 1 | 0.8242 | 0.2031 |
| $i$      | 1 | 0.2529 |

| $k$      | 1 |

<table>
<thead>
<tr>
<th>High Capital Adjustment Costs ($\xi = 0.1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
</tr>
<tr>
<td>$y$</td>
</tr>
<tr>
<td>$u$</td>
</tr>
</tbody>
</table>

Correlation Matrix

| $v$      | 1 | 0.9603 | 0.6524 | 0.0534 |
| $\theta$ | 1 | 0.8073 | 0.1599 |
| $i$      | 1 | 0.2466 |

| $k$      | 1 |

B.2 Importance of Training Costs

In the baseline model, in order to incorporate the empirical feature that firms, when hiring, have to pay costs other than vacancy posting, I departed from standard search models. Here, I show that the results are not only robust to changes in that specification but, as expected, those costs act as a dampening mechanism. As in the previous section, the model is recalibrated to match the same non-stochastic steady state values that were targeted in the baseline case. Table B2 shows the business cycle statistics in the absence of hiring costs.

Table B2: Importance of ‘Start-up’ Costs

<table>
<thead>
<tr>
<th>Model Without Training Costs ($\iota = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
</tr>
<tr>
<td>$y$</td>
</tr>
<tr>
<td>$u$</td>
</tr>
</tbody>
</table>

Correlation Matrix

| $v$      | 1 | 0.9608 | 0.7598 | 0.0345 |
| $\theta$ | 1 | 0.8938 | 0.1321 |
| $i$      | 1 | 0.2336 |

| $k$      | 1 |
Neither the qualitative nor quantitative properties of the model are altered by the addition of training costs. As expected, these costs dampen the response of the aggregates. With start-up costs, the future value associated with having a worker increases since, in the absence of a separation, the firm will not have to pay for those costs in a future period. This is captured by equation (A.19). This means that following a negative shock, firms are more reluctant to reduce the number of workers in the firm, dampening the effects of both credit and TFP shocks. Figure B1 captures these dynamics for both credit and TFP shocks. Consistent the results in previous sections, while the former dampening occurs under both disturbances, the result is significantly more noticeable with respect to changes in credit conditions.
C Search Model Without Financial Frictions

In this section I present a search model in which firms do not face financial constraints. Since the purpose of assuming different discount factors, was to obtain a borrowing constraint that was binding in steady state, this assumption is being dropped so \( \beta_h = \beta_c = \beta \). As in Merz (1995) and Andolfatto (1996), firms in this environment do not have access to an inter-period loan. Those are the only differences with the model presented in the paper.

Households solve the same problem as in the baseline model,

\[
H(\omega_{nfh}^t; \Omega_t^{nf}) = \max_{\{c_t, a_t\}} \{ c_t, a_t \} \\
\ln(c_t) - \varphi n_{h,t} + \mathbb{E}_t \beta H_{t+1}(\omega_{nfh}^{t+1}; \Omega_{t+1}^{nf})
\]

subject to

\[
c_t + a_{t+1} R_t + T_t = w_t n_{h,t} + a_t + u_t s
\]  

and the laws of motion for \( z_t \), which remains unchanged from the model with financial frictions. Now the \( \omega_t^{nfh} = \{n_{h,t-1}\} \) be the vector of individual states for households and \( \Omega_t^{nf} = \{k_t, n_{t-1}; z_{t-1}\} \) be the vector of aggregate states. From this problem the Euler equation is obtained,

\[
\frac{1}{R_t} = \beta_h \mathbb{E}_t \frac{u'(c_{t+1})}{u'(c_t)}
\]  

(C.2)

The problem faced by firms is not given by

\[
J(\omega_{nf,e}^t; \Omega_t^{nf}) = \max_{\{d_t, i_t, k_t\}} \{d_t + \mathbb{E}_t \Lambda_{t+1}^c | \lambda_{t+1}^{nf} \}
\]

subject to

\[
z_t k_t^{\alpha} n_{c,t}^{1-\alpha} = d_t + w_t n_{c,t} + i_t + \nu_t \mu(\theta_t) + \psi(v_t)
\]  

(C.3)

\[
k_{t+1} = (1 - \delta) k_t + \Phi \left( \frac{i_t}{k_t} \right) k_t
\]  

(C.4)

\[
n_{c,t} = (1 - x) n_{c,t-1} + v_t \mu(\theta_t).
\]  

(C.5)

Where, \( \omega_{nf,e}^t = \{k_t, n_{c,t-1}\} \) is the firm’s vector of individual states. As before, when optimizing, the individual firm takes as given the probability that a vacancy will be filled, \( \mu(\theta_t) \), the gross interest rate, \( R_t \), the stochastic discount factor of capitalists, \( \Lambda_{t+1}^c \), and the wage, \( w_t \).

Letting \( \lambda_{1,t}, \lambda_{2,t}, \) and \( \lambda_{3,t} \) denote, respectively, the multipliers on the budget constraint,
(C.3), the capital law of motion, (C.4), and the law of motion for employment, (C.5), the first-order necessary conditions for \(i_t, k_{t+1}, \) and \(v_t\), are given by

\[
\lambda_{2,t} = \frac{1}{\Phi_i} \tag{C.6}
\]

\[
\lambda_{2,t} = E_t A_{t+1|t} \left\{ F_{k,t+1} - \frac{i_t}{k_t} + \mu_{k,t+1} \left[ 1 - \delta + \Phi \left( \frac{i_t}{k_t} \right) \right]\right\} \tag{C.7}
\]

\[
\lambda_{3,t} = \frac{\iota \mu(\theta_t) + \psi'(v_t)}{\mu(\theta_t)} \tag{C.8}
\]

where, as in the main text, \(\Phi_i\) and \(F_k\) represent the first derivatives of the investment adjustment cost function with respect to investment and the marginal product of capital, respectively.

The procedures to obtain both the job-creation and the wage equations follow the steps described previously for the model with financial frictions and it is omitted. The job-creation is in this case,

\[
J_{n,t} = \left[ (1 - \alpha) z_t \left( \frac{k_t}{n_{c,t}} \right)^{\alpha} - w_t \right] + (1 - x) E_t A_{t+1|t} J_{n,t+1}. \tag{C.9}
\]

The wage that solves the generalized Nash bargaining problem is given by

\[
w_t^* = \phi \left[ F_{n,t} + (1 - x) E_t A_{t+1|t} \frac{\iota \mu(\theta_{t+1}) + \psi'(v_{t+1})}{\mu(\theta_{t+1})} \right] + (1 - \phi) \left[ \frac{\varphi}{w'(c_t)} + s \right]
- \phi (1 - x) E_t A_{t+1|t} \left\{ \left[ 1 - f(\theta_{t+1}) \right] \frac{\iota \mu(\theta_{t+1}) + \psi'(v_{t+1})}{\mu(\theta_{t+1})} \right\}. \tag{C.10}
\]

Now that the model was completely described, I can define the equilibrium.

**Definition 2 (Equilibrium Without Financial Frictions)** A recursive equilibrium is defined as a set of i) firm’s policy functions \(d(\omega^{nf,c}; \Omega), n_c(\omega^{nf,c}; \Omega^{nf}), k(\omega^{nf,c}; \Omega^{nf}), i(\omega^{nf,c}; \Omega^{nf}),\) and \(v(\omega^{nf,c}; \Omega^{nf}); ii) household’s policy functions \(c(\omega^{nf,h}; \Omega^{nf}) \) and \(n_h(\omega^{h}; \Omega); iii) a lump sum tax \(T\), iv) prices \(w(\Omega)\) and \(R(\Omega);\) and v) law of motion for the aggregate states, \(\Omega' = \Psi(\Omega),\) such that: i) firms’ policies satisfy conditions (C.6)-(C.8) and (C.9); ii) household’s policy function satisfies (C.2); iii) the wage is determined by (C.10); iv) labor demanded by firms is equal to labor supplied by workers, \(n_c(\omega^{nf,c}; \Omega^{nf}) = n_h(\omega^{nf,h}; \Omega^{nf}); iv) the law of motion \(\psi(\Omega^{nf})\) is consistent with individual decisions and with the stochastic process for \(z\), and vi) the government has a balanced budget so that \(s(1 - n) = T\).
D Enforcement Constraint

As is common in models that derive optimal contracts in the presence of working capital loans, following a default financial intermediaries can confiscate the capital but not the output the firm has produced. In other words, the liquid funds cannot be recovered, only the installed capital. Explicitly, and similar to Hart and Moore (1994) and Perri and Quadrini (2011), I am making the following assumptions,

Assumption 1 *The possibility of default arises at the end of the period, before the intra-period loan is due but after production is observed.*

Assumption 2 *Following a default, and before next period’s capital is incorporated, financial intermediaries confiscate the firm and sell each unit of physical capital at $\eta_t q_{k,t}$.*

Assumption 3 *Following a default, production cannot be seized.*

These assumptions imply that firms are constrained in their ability to borrow. Specifically, the inter-period and intra-period loans are, as in Kiyotaki and Moore (1997), limited by their holdings of capital. However, since liquidation entails a costly process, lenders are only able recuperate a fraction $\eta_t$ of the value of the physical capital stock held by the firm at time $t$. This value is given by $q_{k,t}k_t$, where $q_{k,t}$ represents the (marginal) Tobin’s $Q$ and is formally defined as the ratio of the two Lagrange multipliers $\mu_{k,t}/mu_{c,t}$. I follow Liu et al. (2013) by interpreting $\eta_t$ as an exogenous “collateral shock,” which reflects the uncertainty in the tightness of the credit market. From the lender’s perspective, $\eta_t$ captures the uncertainty with respect to the liquidation value of the firm and its dynamics are represented by the following stochastic process

$$\ln \eta_t = (1 - \rho_\eta) \ln \bar{\eta} + \rho_\eta \ln \eta_{t-1} + \epsilon_{\eta,t}$$

with $\epsilon_{\eta,t} \sim N(0, \sigma_\eta)$, where $\bar{\eta}$ is the mean value of the process and $\rho_\eta$ is its persistence. I further assume that,

Assumption 4 *In the case of default, financial intermediaries have no bargaining power in the debt re-negotiation and they do not value the stock of workers within the firm.*

Now I can state the following proposition, whose derivation is provided in below,

Proposition 1 *Under assumptions 1-4, the following enforcement constraint can be derived as an incentive compatible contract between financial intermediaries and firms:*

$$l_t + \frac{b_{t+1}}{R_t} \leq \eta_t q_{k,t}k_t.$$  \hspace{1cm} (D.1)
Note that the value of the firm at time $t$ can be written as

$$J(\omega^c_t; \Omega_t) = d_t + \mathbb{E}_t \Lambda^c_{t+1|t} J(\omega^c_{t+1}; \Omega_{t+1})$$

From Assumption 1 the value of not defaulting, $v^{f,n}$ is

$$v^{f,n} = \mathbb{E}_t \Lambda^c_{t+1|t} J_{t+1} (\omega^c_{t+1}; \Omega_{t+1})$$

In the case of default both firms and lenders start negotiations. If an agreement is reached, firms agree to pay lenders a quantity $\nu_t$ the continuation value of the firm in case of a successful negotiation, $v^{f,s}$, can be expressed as

$$v^{f,s} = \mathbb{E}_t \Lambda^c_{t+1|t} J_{t+1} (\omega^c_{t+1}; \Omega_{t+1}) + l_t - \nu_t$$

From Assumption 3, it follows that the value for the firm of an unsuccessful negotiation, $v^{f,u}$, is

$$v^{f,u} = l_t$$

Consequently, from the perspective of the firm the net value of an agreement, $v^{f,net}$

$$v^{f,net} = v^{f,u} - v^{f,s} = \mathbb{E}_t \Lambda^c_{t+1|t} J_{t+1} (\omega^c_{t+1}; \Omega_{t+1}) - \nu_t$$

The value of the lender of a successful negotiation for the lender, $v^{l,s}$, is, on the other hand,

$$v^{l,s} = \nu_t + \frac{b_{t+1}}{R_t}$$

If the agreement is not reached from assumptions 2 and 4, the value of unsuccessful negotiations, $v^{f,u}$ is

$$v^{f,u} = \eta_t q_{k,t} k_t$$

Therefore, I define the net value of renegotiation from the lender’s perspective, $v^{l,net}$, as

$$v^{l,net} = v^{l,u} - v^{l,s} = \nu_t + \frac{b_{t+1}}{R_t} - \eta_t q_{k,t} k_t$$
The joint surplus of renegotiation, \( V(\omega_{t+1}; \Omega_{t+1}) \), is the sum of the net value of renegotiation for both parties. Formally,

\[
V(\omega_{t+1}; \Omega_{t+1}) = v_{t+1} + v_{t+1}^{\text{net}}
= \mathbb{E}_t \Lambda_{t+1|t}^c J_{t+1} (\omega_{t+1}; \Omega_{t+1}) + \frac{b_{t+1}}{R_t} - \eta_t k_t
\]

From assumption 4, in case of default the firm gets its liquidity plus the joint surplus of renegotiating the debt. The value of default, \( v_{t+1}^{\text{f,d}} \), is then

\[
v_{t+1}^{\text{f,d}} = l_t + V(\omega_{t+1}; \Omega_{t+1})
= \mathbb{E}_t \Lambda_{t+1|t}^c J_{t+1} (\omega_{t+1}; \Omega_{t+1}) + l_t + \frac{b_{t+1}}{R_t} - \eta_t q_{k_t} k_t
\]

In order to rule out default it is needed that the value of not defaulting is at least as large as the value of defaulting,

\[
v_{t+1}^{\text{f,n}} \geq v_{t+1}^{\text{f,d}}
= \mathbb{E}_t \Lambda_{t+1|t}^c J_{t+1} (\omega_{t+1}; \Omega_{t+1}) + l_t + \frac{b_{t+1}}{R_t} - \eta_t q_{k_t} k_t
\]

After arranging terms equation (D.1) in the main text is obtained:

\[
l_t + \frac{b_{t+1}}{R_t} \leq \eta t q_{k_t} k_t
\]

### E Data Description and Sources

To construct measures for output, \( y_t \) and investment, \( i_t \), I use quarterly data extracted from the National Income and Product Accounts (NIPA) Tables of the Bureau of Economic Analysis (BEA). Output is nominal gross domestic product (Series A191RC1) divided by its deflator (Series A191RD3). Investment is in turn defined as ‘personal consumption expenditures in durable goods’ (Series DDURRC1) plus the ‘nonresidential’ and ‘residential’ items of ‘gross private domestic investment’ (Series A008RC1 and A011RC1, respectively); each series is then divided by their respective deflator (Series DDURRD3, A008RD3, and A011RD3). Nominal series are seasonally adjusted at annual rates taken from Table 1.5.5. Price deflators are extracted from Table 1.1.9. In order to create per capita series, and with data obtained from the Current Population Survey (CPS) of the Bureau of Labor
Statistics (BLS), nominal variables are divided by the quarterly average of a monthly series of civilian noninstitutional population of ages 16 to 64 (Series LNS10000000 minus Series LNU00000097). For the physical capital stock, $k$, I integrate the data series used in Fernald (2009), available at John Fernald’s website.

Vacancies in this paper are constructed using the method proposed by Barnichon (2010). The method combines job openings from the JOLTS data set (Series JTS00000000JOL), the Help-Wanted Online Advertisement Index published by the Conference Board (Series HWOL), and the Help-Wanted Print Advertising Index that was discontinued in October 2008 and it was also constructed by the Conference Board. Unemployment, $u$, is the quarterly average of the monthly seasonally adjusted unemployment rate reported by the BLS (Series LNU04000000). Wages, $w$, are created by dividing the ‘average hourly earnings of production workers in the private sector’ from the BLS (Series CES0500000008) over the consumer price index (Series CUSR0000SA0), also available from the BLS.

The measure of debt used in the paper comes from the Flow of Funds Accounts of the United States (FOF) available from the Board of Governors of the Federal Reserve System. Specifically, I use the quarterly seasonally adjusted ‘credit market debt outstanding’ (D.3) in the ‘nonfinancial corporate business sector’ (Line 6) (Series LA144104005.Q). The debt-to-GDP ratio, $b/y$, was created by dividing ‘credit market debt outstanding’ by the nominal gross domestic output.