Optimal Monetary Policy and Imperfect Financial Markets: A Case for Negative Nominal Interest Rates?*

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Abstract

This paper studies optimal monetary policy in a model with credit frictions and money demand. We show that augmenting a standard New-Keynesian model with money demand and financial frictions generates a mechanism that, in equilibrium, gives rise to optimal negative nominal interest rates. In addition, we find that the tighter credit markets are, the lower the optimal nominal policy interest rate and the more likely is to be negative. Quantitatively, when credit constraints are binding, a standard calibration of the model generates an optimal nominal policy interest rate that is roughly -4% annually.

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1 Introduction

Following the recent global financial crisis, central banks around the world cut their nominal interest rates to roughly zero in an attempt to stimulate economic activity. A near zero nominal interest rate prevented central banks from using this policy instrument any further, thus leading to the adoption of unconventional monetary policy schemes. Moreover, in Europe, this situation has started a recent debate about the possibility of moving to negative deposit rates. Motivated by this atypical state of affairs, this paper studies optimal monetary policy in an environment characterized by credit frictions that limit the borrowing of households who also value holding real money balances. In particular, we investigate whether in this environment optimal monetary policy provides a theoretical justification for adopting a negative nominal interest rate.

Because of its apparent empirical implausibility, economic theory has often disregarded the study of negative nominal interest rates. The reason for that is straightforward: if the nominal interest rate is negative, agents will be better off holding cash than bank deposits. Moreover, negative interest rates may endanger the acceptance of currency as legal tender and give rise to alternative media of exchange. These considerations could perhaps explain the choice of, for instance, the Bank of Japan during the 1990s and the Federal Reserve since 2008 to not set interest rates below zero. However, as part of the search for effective monetary policies following the recent economic crisis, a few banks in Europe, such as Denmark, Germany, Switzerland and the European Central Bank (ECB, henceforth), have recently moved to negative deposit interest rates.[1] The present study suggests one mechanism, built into a standard New Keynesian model with money demand augmented to incorporate borrowing constraints, that justifies the optimality of negative nominal interest rates.

We present a model in which households derive direct utility from money holdings and face a constraint that limits their ability to borrow. We show that, in this environment, a negative nominal policy interest rate emerges as optimal. Quantitatively, the nominal policy interest rate is roughly -4% annually. We also find that the tighter the credit conditions are, the more negative the nominal policy interest rate. Under a reasonable parametrization of the model, the tightness of the credit constraint is sufficient to cause the monetary authority to optimally set a negative nominal interest rate.

Our work is related to the limited literature on negative nominal interest rates and to

[1] See for instance, S. Riecher and J. Black, "Draghi Takes ECB Deposit Rate Negative in Historic Move", *Bloomberg*, June 5, 2014, http://bloom.bg/1oYAZDR; Weing, J., "Sweden Imposes Negative Interest Rate and Plans Bond-Buying Program", *The New York Times*, February, 2015, http://nyti.ms/1J03w6B; Duxbury, C., "Denmark's Top Banker Extols Rates Policy", *Wall Street Journal*, September 3, 2013, http://goo.gl/whvJRU; and Mankiw, G. N., "It May be Time for the Fed to go Negative", *The New York Times*, April 18, 2009, http://goo.gl/hoK21m.

the literature that has studied optimal monetary policies. A recent example of a study that addresses the former is Buiter (2009), who studied the effects of paying negative interests on currency as one way to overcome the zero lower bound on nominal interest rates.[2] Regarding the literature on optimal monetary policy, the seminal work of Schmitt-Grohe and Uribe (2004) showed that in a model with sticky prices, imperfect product markets and money demand, optimal monetary policy calls for setting a positive nominal interest rate, implying that the nominal distortion dominates the monetary distortion. Chugh (2006) arrived to a similar conclusion in a model that includes sticky nominal wages (with and without sticky prices). These papers, however, abstract from credit frictions. On the other hand, the studies that focus on optimal monetary policy and account for the presence of imperfect financial markets assume no role for money (examples are Monacelli (2006), Curdia and Woodford (2009), and Carlstrom et al. (2010)).[3] In our study, we show that a standard calibration of a model that combines money demand, sticky prices and borrowing constraints on households can quantitatively account for a negative optimal nominal interest rate.

The intuition for our result is straightforward: in an environment without borrowing constraints on households, the economy is satiated with real money balances when the nominal interest rate is zero (the well known "Friedman Rule"). With a borrowing constraint on households, however, we show that satiating the economy with real money requires a negative nominal interest rate. Put differently, in this paper, there are three channels operating on the optimal nominal interest rate: the monetary distortion (which calls for a zero nominal interest rate), the nominal distortion (which calls for a positive nominal interest rate) and the credit friction (which calls for a negative nominal interest rate). We show that the credit friction dominates the other two forces, thus leading to a negative nominal interest rate.

In a robustness exercise, we model the intermediation sector as an imperfectly competitive market. We do so by assuming monopolistic competition among lenders, who, given the nominal policy interest rate, optimally set their loan and deposit nominal interest rates. By having monopolistic power in the intermediation sector, the model's extension allows for deviations between the loan rate and the policy rate and, hence, it is able to generate a positive loan rate even if the policy rate is negative. We find that the sign of the nominal

[2]See Ilgmann and Menner (2011) for a review of the literature on negative nominal interest rates.

[3]As in the present paper, Monacelli (2006) studies the optimal monetary policy in an environment in which households are constrained, but abstracts from money demand. Curdia and Woodford (2009) account for a credit spread between the saving interest rate and the borrowing interest rate and show that variations in the spread over time have greater consequences for the relation between the policy rate and aggregate expenditure and for the relation between real activity and the inflation rate. More recently, Carlstrom et al. (2010) introduce agency costs in an otherwise standard New Keynesian model and show that the welfare criterion is modified to account for the tightness of the credit markets and that those agency costs manifest themselves as markup shocks in the Phillips curve.

policy interest rate depends on the spread between the policy interest rate and the loan rate. If that spread is larger than the loan rate, then the nominal policy interest rate will be negative. More specifically, the policy interest rate is surely negative if the loan rate is nonpositive and, depending on the degree of market power of lenders, it may be negative even if the loan rate is positive. Quantitatively, the optimal policy interest rate varies between -2% and -4% annually.

The remainder of the paper proceeds as follows. Section 2 outlines the model and defines the private-sector equilibrium and the optimal monetary policy problem. The calibration and the main quantitative results about the optimal interest rate policy are presented in Section 3. The model with market power in the banking sector is presented in Section 4. Section 5 concludes.

2 The Model Economy

Our economy is populated by two types of agents: households and entrepreneurs. Households derive utility from consumption of the final good, leisure, and cash holdings. They provide labor to intermediate-good firms which are owned by risk-neutral entrepreneurs. Besides the labor market, these agents also interact in the financial market via banks that are owned exclusively by households. However, we assume that, due to a problem of imperfect enforcement, households are constrained in their ability to borrow. Nonneutrality of money arises because it is costly for intermediate-good firms to adjust their prices. We start by describing the economic agents and then turn to discuss the interplay between the credit conditions and the optimal nominal policy interest rate.

2.1 Households

Households maximize lifetime expected utility over consumption, c_t, hours, n_t, and real cash holdings, m_t. Households borrow b_t at a gross interest rate of R_t, which is to be paid next period. However, as in Kiyotaki and Moore (1997), the presence of financial frictions that arise from a limited enforcement problem limits their borrowing. Since the model does not feature capital, wages will be the collateralizable asset. In particular, we assume that borrowing is limited to a fraction η of households' labor income, $w_t n_t$, with w_t being the real wage.[4] This enforcement constraint can be derived as the optimal solution to a debt renegotiation problem as in, for instance, Perri and Quadrini (2014).

[4]Tying borrowing to labor income is common in the literature, which is consistent with the lending practices of banks (Ludvigson, 1999). See also Marcet and Singleton (1999).

Formally, the problem of households is given by:

$$\max_{\{c_t,n_t,m_t,b_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t, m_t) \tag{1}$$

subject to the sequence of period budget constraints and borrowing constraints:

$$c_t + m_t + \frac{R_{t-1} b_{t-1}}{\pi_t} = \frac{m_{t-1}}{\pi_t} + b_t + w_t n_t + \Pi_t \tag{2}$$

$$b_t \leq \eta w_t n_t. \tag{3}$$

where β is the subjective discount factor and $u(c_t, n_t, m_t)$ is the period utility function that satisfies the following properties: $\partial u/\partial c > 0$, $\partial^2 u/\partial c^2 < 0$, $\partial u/\partial m > 0$, $\partial^2 u/\partial m^2 < 0$, and $\partial u/\partial n < 0$, $\partial^2 u/\partial n^2 < 0$. In addition, $\pi_t = P_t/P_{t-1}$ is the time t gross inflation rate, P_t is the aggregate price level, and Π_t denotes the profits from owning the final-good firms.

After substituting for the optimality condition with respect to consumption, the first order conditions with respect to labor, bonds and cash holdings yield:

$$-\frac{u_{n,t}}{u_{c,t}} = w_t(1 + \eta \mu_{b,t}) \tag{4}$$

$$u_{c,t} = \beta R_t \mathbb{E}_t \left(\frac{u_{c,t+1}}{\pi_{t+1}} \right) + \mu_{b,t} u_{c,t} \tag{5}$$

$$u_{m,t} = u_{c,t} \left[\frac{(R_t - 1) + \mu_{b,t}}{R_t} \right] \tag{6}$$

where $\mu_{c,t}$ and $\mu_{c,t}\mu_{b,t}$ are, respectively, the Lagrange multipliers on the budget and the borrowing constraints.

Equations (4) and (5) are, respectively, the labor supply condition and the consumption Euler condition, both augmented by the existence of the credit constraint. Equation (6) is the real money demand condition; it relates the marginal utility of consumption, the marginal utility of money, the nominal interest rate and the tightness of the credit constraint. Note that the money demand condition in this paper differs from the standard money demand condition due to the existence of the credit friction. In what follows, this expression will be key in determining the sign of the optimal nominal policy interest rate.

2.2 The Production Sector

As is standard in the literature, two types of firms operate in this sector: monopolistically competitive intermediate-good firms who produce differentiated products and perfectly competitive firms who transform intermediate goods into final goods using a constant return to scale technology. Each intermediate-good firm faces an adjustment cost for its prices, which is the source of price rigidity in this model.

2.2.1 Final-Good Firms

Firms in this sector assemble a continuum of intermediate goods into final goods using the technology:

$$y_t = \left(\int_0^1 y_{j,t}^{\frac{\epsilon-1}{\epsilon}} \, dj \right)^{\frac{\epsilon}{\epsilon-1}} \tag{7}$$

where $y_{j,t}$ is the output of firm j at period t and $\epsilon > 1$ is the elasticity of substitution between two varieties of final goods. In line with standard Dixit-Stiglitz based New Keynesian models, maximization of profits yields the following downward-sloping demand function for the variety j:

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\epsilon} y_t \tag{8}$$

where $P_t = \left(\int_0^1 P_{j,t}^{1-\epsilon} \, dj \right)^{\frac{1}{1-\epsilon}}$ is the corresponding Dixit-Stiglitz aggregate price index.

2.2.2 Intermediate-Good Firms

There is a continuum of measure one of monopolistically competitive firms indexed by j, that are managed/owned by an entrepreneur. Each period, an entrepreneur, who discounts future profits at the rate δ, have access to a riskless financial asset, $d_{j,t}$, that pays an interest rate of R_t. Since the purpose is to focus on the credit constraints that affect households, we assume that households are relatively less patient, i.e. $\beta < \delta$. As is standard in the literature, this implies that in the steady state the credit constraint will be binding and households' borrowing originate from entrepreneurs' deposits (e.g. through a perfectly-competitive intermediation sector). We assume that the utility function of each entrepreneur is linear and, hence, equal to the profit function of the corresponding intermediate-good firm. Consequently, we will refer to this type of agents as intermediate-good firms, entrepreneurs, or depositors, interchangeably.

The intermediate-good firm hires labor to produce a differentiated product using the following technology:

$$y_{j,t} = z_t n_{j,t}^{\alpha} \tag{9}$$

with z_t denoting total factor productivity (TFP) and α being the elasticity of output with respect to labor. As in Rotemberg (1982), the pricing of each firm is subject to a quadratic adjustment cost expressed in units of the final good.

Each firm j chooses its price, deposits, and labor to maximize the expected present discounted stream of profits:

$$\max_{\{n_{j,t},d_{j,t},P_{j,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \Omega_{t|0} \left[\frac{P_{j,t}}{P_t} y_{j,t} - w_t n_{j,t} - \frac{\varphi}{2} \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2 y_t + \frac{R_{t-1} d_{j,t-1}}{\pi_t} - d_{j,t} \right] \tag{10}$$

subject to the downward-sloping demand function for its product given by equation (8). Note that, since entrepreneurs own those firms, their future profits are discounted by their stochastic discount factor $\Omega_{t+k|t}$ which, since entrepreneurs are risk-neutral, is equal to δ.

The Lagrange multiplier on the output constraint, $mc_{j,t}$, measures the contribution of one additional unit of output to the revenue of the intermediate-good firm, and, in equilibrium, it equals the real marginal cost of the firm. We focus on a symmetric equilibrium in which all firms set the same price and choose the same amounts of labor and deposits. Profit maximization then yields the following standard labor demand condition:

$$mc_t = \frac{w_t}{\alpha z_t n_t^{\alpha-1}}. \tag{11}$$

Rotemberg pricing gives the standard forward-looking price Phillips curve:

$$\epsilon(1 - \alpha mc_t) = 1 - \varphi(\pi_t - 1)\pi_t + \delta\varphi\,\mathbb{E}_t \left[(\pi_{t+1} - 1)\pi_{t+1} \frac{y_{t+1}}{y_t} \right]. \tag{12}$$

This equation shows that the current price inflation rate is a function of the expected price inflation rate and the current real marginal cost.

Finally, the optimal choice of deposits yields:

$$1 = \delta R_t\,\mathbb{E}_t \left(\frac{1}{\pi_{t+1}} \right). \tag{13}$$

To determine the shadow value of the households's borrowing at the steady state, we can combine equation (13) with the demand for loans by households, equation (5). This gives:

$$\mu_b = \frac{\delta - \beta}{\delta}. \tag{14}$$

Therefore, the borrowing constraint will bind at the steady state if $\delta > \beta$, as is the case in this model. As the gap between the two discount factors increases, the steady-state borrowing constraint becomes tighter.[5] [6]

[5]This is a standard result models with credit constraints; see, for instance, Monacelli (2009).

[6]Conditions (11)-(13) will also hold if we assume a setup with four agents: households, depositors who are more patient that households and have a linear utility in consumption, intermediate-good firms that are owned by depositors and final-good firms. The above setup simplifies the model by incorporating depositors and intermediate-good producers into one group: the above problem can be viewed as maximizing utility

2.3 Nominal Interest Rate and Credit Conditions

To see the effects of the credit constraint on the sign of the nominal policy interest rate, consider two alternative cases. First, if the economy is satiated with real money balances, then the marginal utility of cash holding is zero, $u_{m,t} = 0$. In this case, condition (6) yields:

$$i_t = -\mu_{b,t} \tag{15}$$

implying that the net nominal policy interest rate will be negative as long as the borrowing constraint is binding ($\mu_{b,t} > 0$). Therefore, if the credit constraint is binding, full satiation of the economy with real money balances happens only when the nominal interest rate is negative. Combining the deterministic steady state version of this condition with condition (37) gives $i = \frac{\beta-\delta}{\delta}$. Therefore, when $u_m = 0$, the value of the nominal interest in the deterministic steady state is determined by the gap between the two subjective discount factors.

Second, if $u_{m,t} > 0$, then i_t may be zero, positive, or negative. This can be determined only by solving the model quantitatively, but the likelihood of a negative nominal policy interest rate increases as credit conditions become tighter. Put differently, setting a negative nominal policy interest rate is more warranted when the credit conditions faced by households worsen.

2.4 Market Clearing and Equilibrium

In equilibrium, the resource constraint of the economy is given by:

$$y_t = c_t + \frac{\varphi}{2}\left(\pi_t - 1\right)^2 y_t \tag{16}$$

and the loan market clears:

$$b_t = d_t. \tag{17}$$

Definition 1 (Private-Sector Equilibrium) *Given the exogenous processes for z_t and R_t, the private-sector equilibrium is a sequence of allocations $\{c_t, d_t, b_t, m_t, mc_t, n_t, w_t, y_t, \pi_t, \mu_{b,t}\}$ that satisfy the equilibrium conditions (3)-(6), (9), (11)-(13) and (16)-(17).*

2.5 The Optimal Monetary Policy Problem

Our main analysis assumes that the monetary authority solves a Ramsey-type commitment problem: it chooses allocations to maximize the lifetime utility of households subject

from consumption subject to the budget constraint $\frac{P_{j,t}}{P_t}y_{j,t} - w_t n_{j,t} - \frac{\varphi}{2}\left(\frac{P_{j,t}}{P_{j,t-1}} - 1\right)^2 y_t + \frac{R_{i,t-1}d_{j,t-1}}{\pi_t} - d_{j,t} = c^d_{j,t}$, with $c^d_{j,t}$ being the consumption of depositor j. By substituting for $c^d_{j,t}$ in the objective function, we get (10).

to the resource constraint and the private-sector equilibrium conditions. In an extension, we will also consider the discretionary case, where the central bank optimizes period by period.

Definition 2 (Optimal Policy Problem) *Given the exogenous process of z_t, the monetary authority chooses a sequence of allocations $\{c_t, d_t, b_t, m_t, mc_t, n_t, R_t, w_t, y_t, \pi_t, \mu_{b,t}\}$ to maximize (1) subject to the equilibrium conditions (3)-(6), (9), (11)-(13) and (16)-(17).*

3 Quantitative Exercise

This section presents the functional forms, discusses the parameterization of the model, and then shows the main quantitative results about the optimal interest rate policy.

3.1 Functional Forms and Parameterization

We assume the following period utility function for households:

$$u(c_t, n_t, m_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\gamma}}{1+\gamma} + \mu \frac{m_t^{1-\tau}}{1-\tau} \tag{18}$$

where σ is the curvature parameter of the period utility function of consumption, τ is the curvature parameter of the period utility function of real money, γ is the inverse of the intertemporal elasticity of substitution of labor, and χ and μ are scaling parameters.

Total factor productivity evolves according to the following AR(1) process:

$$\log z_t = (1 - \rho_z) \log z + \rho_z \log z_{t-1} + u_t \tag{19}$$

with ρ_z being the AR(1) coefficient of the TFP process, $z = 1$, and $u_t \sim \mathcal{N}(0, \sigma_u^2)$.

Table 1 summarizes the benchmark parameterization of the model. The values chosen for the labor share of output, the consumption curvature parameter, and the elasticity of substitution between differentiated products are fairly standard in the literature. The households' subjective discount factor is set to 0.94, in line with Carlstrom and Fuerst (1997). The value of γ implies a labor supply elasticity of 4, which is within the range of the usually assumed values of the labor supply elasticity in macroeconomic models (see Peterman (2012)). The disutility of labor is chosen so that households work 21% of their total time endowment (the equivalent to approximately 35 hours per week). The value of τ implies an interest-elasticity of money demand of about -0.16, which is the elasticity of money demand with respect to the Three-Month Treasury Bill over the period 1964:Q2–2007:Q4.[7]

[7] Our sample ends in 2007:Q4 to avoid any bias in the estimation of the interest elasticity of money

Following Monacelli (2006), we set the loan-to-income ratio to 0.75, however, in our robustness exercise we will consider other values for this parameter. As is standard, the value of the price rigidity parameter, φ, it is obtained by making the slope of the Phillips curve under Rotemberg (1982) pricing being equal to the slope of the Phillips curve under Calvo (1983) pricing and it implies a price duration of two quarters.[8] Finally, the standard deviation of the productivity shock is chosen so that the unconditional standard deviation of the cyclical component of output in our model matches its empirical counterpart for the U.S.[9]

Table 1: Values of the Parameters

Parameter	Description	Value
β	Households' discount factor	0.94
σ	Consumption curvature parameter	1.00
χ	Disutility of labor	3.86 3
γ	Inverse of labor supply elasticity	0.25
τ	Money curvature parameter	6.21
η	Loan-to-value ratio	0.75
α	Labor share of output	2/3
δ	Depositors' discount factor	0.99
φ	Price adjustment cost	9.90
ϵ	Elasticity of subs. between goods	6.00
ρ_z	AR(1) coefficient of TFP	0.95
σ_u	Std. Dev. to the TFP shock	0.033

3.2 Quantitative Results

Table 2 presents the means and standard deviations of the nominal policy interest rate, the inflation rate, and output for four different cases: i) money demand only, i.e. flexible prices, perfect product markets, and frictionless credit markets; ii) sticky prices and monopolistic competition only, i.e. model with neither money demand nor credit frictions; iii) sticky prices, monopolistic competition, money demand, but no credit frictions; and iv), the full model, i.e. money demand, sticky prices, monopolistic competition, and credit frictions. Both the interest and the inflation rate are presented in annualized percentage terms.

Consider first the case with money demand but no sticky prices, no monopolistic competition in the product market and no credit frictions. As expected, optimal monetary policy calls for implementing the Friedman Rule- the nominal interest rate is zero. The correspond-

demand following the dramatic decline in the nominal interest rate and the sharp increase in M1 since 2008.

[8] This approach follows Faia and Monacelli (2007) and Monacelli (2009), among others.

[9] As is standard, the volatility of the time series is obtained by calculating its log-deviations from an HP trend with smoothing parameter of 1,600; in the data this gives as a value of 0.015.

ing optimal inflation rate is negative and approximately equals to the (annual) rate of time preference. As in Schmitt-Grohe and Uribe (2004) and Chugh (2006), among others, the planner sets a constant path for the nominal interest rate.

When money is not valued and agents interact in a market with sticky prices and monopolistic competition, the planner aims for minimizing the nominal distortion that stems from the cost of adjusting prices. This gives rise to zero inflation rate optimally at all dates and states. Clearly, the nominal interest rate in this is case positive. The introduction of money demand does not change the latter result and the optimal interest rate is still positive (albeit it is lower than in the case with no money demand). Qualitatively, this result is in line with the findings of Schmitt-Grohe and Uribe (2004) and Chugh (2006). Basically, the tension between the nominal and the monetary distortions is largely resolved in favour of the nominal distortion. This is well illustrated in the fact that the optimal nominal interest rate is closer to the value in the second panel of the table than to zero.

Table 2: Optimal Monetary Policy Under Commitment

Case	Variable	Mean	Std. Dev.
Money Demand Only	i	0.0000	0.0000
	π	-3.9633	0.0224
	y	0.3524	0.0150
Sticky Prices and Monopolistic Competition	i	4.1140	0.0210
	π	0.0000	0.0000
	y	0.3520	0.0150
Money Demand, Sticky Prices, and Monopolistic Competition	i	2.4946	0.1891
	π	-1.2982	0.2063
	y	0.3568	0.0150
Full Model	i	-3.7569	0.0058
	π	-7.5495	0.0055
	y	0.3534	0.0150

Numerical results obtained from solving the model using a second-order approximation around the non-stochastic steady state for the Ramsey problem. Moments reported are log-detrended simulations using an HP filter with a smoothing parameter of 1,600. 'Full model' refers to the model with money demand, sticky prices, monopolistic competition and credit frictions. The means of the interest rate and the inflation rate are presented in annualized percentage terms.

The main finding of this paper, however, is presented in the last panel of Table 2. With credit frictions, the planner faces three competing forces: the monetary distortion, the nominal distortion, and the credit distortion. Our results show that this tension is resolved in "favor" of the credit distortion, which results here in a negative nominal interest rate. Note that the results in the third panel can be seen as a particular case of the full model when the borrowing constraint is not binding and they are consistent with our discussion in Subsection 2.3: the more binding the credit constraint is, the lower the optimal nominal interest rate.

Although it may appear counterintuitive that firms are willing to deposit their funds and earn negative nominal interest rates, note that while the *nominal* deposit rate is negative, the *real* rate is positive as the inflation rate is smaller than the nominal rate. Basically, the willingness to hold money for the next period (and thus consume more in the future) clashes with the willingness to borrow (and thus consume more today). To induce more money holdings by the agent, the planner ensures that money is offering a high enough real return, which can happen through strong deflation. This way, despite the negative nominal interest rate, the real interest rate is strongly positive (roughly 4% in the benchmark calibration of the model).[10]

A natural question is whether the qualitative results are driven by the fact that the monetary authority solves a commitment problem. The answer is no. Table 3 presents the results obtained when monetary policy is assumed to be discretionary. Here we solve the full model with a central bank that, every period, solves the one period problem by maximizing the welfare function subject to the equilibrium conditions and the resource constraint. Since the purpose of the exercise is to quantitatively examine how the main result would differ from the commitment case, the parameters of the model remain the same as in Table 1.

With discretion, the optimal nominal interest remains negative, but it is higher than under commitment. This result reflects the so-called "inflation bias" problem with discretionary policy. Since the monetary authority cannot manipulate the inflation expectations of the private sector (as it takes them as given), it sets a higher inflation rate and, consequently, a higher nominal interest rate than under commitment. Comparison of the results with commitment and discretion reveal that the "inflation bias" is nearly 3.5%, which is approximately the difference between the optimal interest rates under both environments.

Table 3: Optimal Monetary Policy Under Discretion – Full Model

Variable	Mean	Std. Dev.
i	-0.1012	0.0010
π	-4.0376	0.0010
y	0.4498	0.0150

Numerical results obtained from solving the model using a second-order approximation around the non-stochastic steady state for the Ramsey problem. Moments reported are log-detrended simulations using an HP filter with a smoothing parameter of 1,600. 'Full model' refers to the model with money demand, sticky prices, monopolistic competition and credit frictions. The means of the interest rate and the inflation rate are presented in annualized percentage terms.

Our results has similarities with Billi (2011), who analyzed the optimal inflation rate, and the corresponding nominal interest rate, in a New-Keynesian model with the ZLB. The

[10] Also, to follow as closely as possible the basic New Keynesian model, we abstract from physical capital accumulation and therefore these deposits are the only technology available to firms to transfer resources across periods.

author shows that the optimal discretionary policy sets a clearly higher nominal interest rate than the optimal policy under commitment. Billi (2011) attributes that to the "fear of deflation" on the part of policy makers and their inability to influence the private-sector expectations. In our model, the combination of money demand and borrowing constraint pushes the nominal interest to be around zero or negative, but qualitatively, the result is similar: with discretion, policy makers choose higher inflation rates and consequently higher nominal interest rates than with commitment.

3.3 Robustness Exercises

3.3.1 Labor Supply Elasticity and Loan-to-Value Ratio

In this section, we start by studying the effects of changes in the labor supply elasticity. In particular, we consider the case with a low labor supply elasticity ($\gamma = 4$) and an infinite labor supply elasticity ($\gamma = 0$). As shown in Table 4, the main result in the paper is robust to different parameterizations for the labor supply elasticity. The optimal interest rate set by the central bank is negative in a model with a low labor supply elasticity and in a model in which all the fluctuations occur at the extensive margin. Quantitatively, the optimal nominal policy interest rate is more negative when the labor supply elasticity is higher.

Table 4: Labor Supply and Collateral Requirement

	Inverse of Labor Supply Elasticity				Loan-to-Value Ratio			
	$\gamma = 4.0$		$\gamma = 0.0$		$\eta = 0.5$		$\eta = 1.0$	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
i	-3.0642	0.0054	-4.0200	0.0060	-3.8306	0.0058	-3.6795	0.0057
π	-6.8841	0.0055	-7.8022	0.0058	-7.6202	0.0056	-7.4751	0.0054
y	0.3533	0.0150	0.3530	0.0150	0.3536	0.0150	0.3533	0.0150

Notes: see Table 2. These results are obtained for the full model with commitment.

Since the degree to which the credit constraint is binding is important for the results of this paper, we also consider different values for the loan-to-value ratio, η. Our key result is also robust to changes to the degree at which households can borrow against their labor income. A lower loan-to-value ratio implies that households are able to borrow a smaller fraction of their labor income and, *ceteris paribus*, it implies a tighter borrowing constraint. Consistent with our previous discussion, a tighter constraint is associated with a lower nominal policy interest rate. [11]

[11] We also study how credit shocks affect the optimal policy interest rate by allowing η to vary. Since our analysis focus on the steady state, we find that adding credit shocks does not alter the key results of this

3.3.2 Linear Production Function

We next show the results when the production function is linear ($\alpha = 1$), which is the standard assumption of the New Keynesian model. As Table 5 shows, our main result is very robust to the assumption about the production technology; in fact, there is a slight decrease in the nominal interest rate and the corresponding inflation rate. More importantly, the optimal annual nominal interest rate continues to be nearly -4% in the benchmark calibration of the model economy.

Table 5: Optimal Monetary Policy- Full Model with Linear Technology

Variable	Mean	Std. Dev.
i	-4.0475	0.0103
π	-7.8346	0.0101
y	0.2096	0.0150

Notes: see Table 2. These results are obtained for the full model with commitment.

3.3.3 Relative Impatience

As discussed in Subsection 2.3, the value of the optimal nominal interest rate depends on the tightness of the credit constraint which, in turn, depends on the difference between the discount factors of both agents ($\delta - \beta$). In this section we seek to find the threshold of β for which the optimal interest rate switches sign. We perform this experiment by focusing on the behavior of the nominal interest rate as β (households' discount factor) gets closer to δ (firms' discount factor), leaving δ constant.

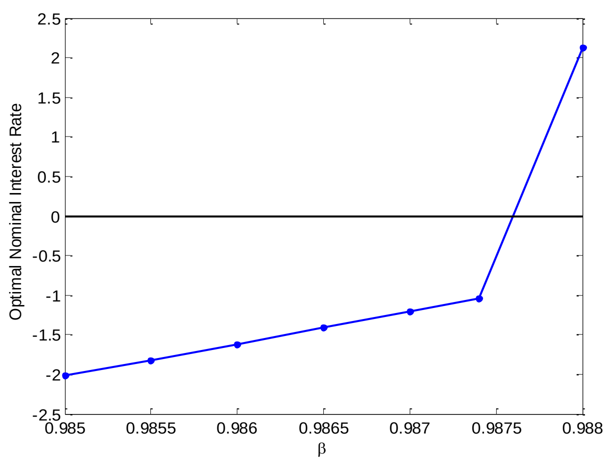

Figure 1: Optimal nominal interest rate in annualized percentage terms

paper as long as the mean of η in the stochastic process is the same as in the baseline calibration

The results are summarized in Figure 1. The optimal nominal interest rate remains negative in the beginning of this range but, as expected, as β increases the optimal interest rates becomes less negative as the shadow value on the credit constraint declines. For β sufficiently close to δ ($\approx \beta = 0.988$), the optimal interest rate becomes positive.

3.3.4 The Model with Cash in Advance Constraint

In Appendix A we consider a version of the paper with a cash in advance (CIA) constraint as an alternative approach for introducing a money demand motive. We show, analytically, that the sign of the nominal policy interest rate depends on the relative importance of the shadow value on the borrowing constraint and the shadow value on the CIA constraint. Consistent with our previous results, the optimal nominal policy interest rate is more likely to be negative when the credit constraint is tighter. Specifically, the optimal nominal interest rate will be negative if the credit constraint is tighter than the CIA constraint. In other words, if the credit friction is more "dominant" than the CIA constraint, the nominal policy rate will be negative. In addition, if monetary policy aims at relaxing the CIA constraint (or relatively relaxing it), then, ceteris paribus, the optimal nominal policy interest rate will be negative.[12,13]

4 Model with Monopolistic Banking Sector

In this section we extend the basic model and, in order to incorporate the empirical feature that the intermediation sector is characterized by imperfect competition and the existence of a spread between the lending and deposit rates, we assume that banks operate in an environment of monopolistic competition and hence have market power in both the loan and the deposit markets.[14] Denoting by R_t^l and R_t^d the lending and deposit rates, respectively, the sequence of budget constraints for households now reads:

[12] Relaxing the CIA constraint in this setup is equivalent to letting $u_{m,t}$ go to zero in our benchmark model.

[13] To avoid departing from a standard New-Keynesian framework, we abstract from incorporating search frictions to motivate the demand for money. Moreover, we can think of the MIU approach as capturing, in a reduced form sense, the outcome of a richer environment in which the demand from money emerges as a equilibrium solution to a double coincidence of wants problem (see for instance Howitt (2003)). It is hard to speculate what would happen to our results if we were to microfound the demand for money as money search models can give rise to equilibrium multiplicity (Burdett and Judd (1983)) which implies that the optimal interest rate is likely not to be unique.

[14] Several studies have focused on analyzing the market power in the banking industry; see for instance Hannan (1991), Demirguc et al. (2003), Patti and Dell'Ariccia (2004) and Mandelman (2010), and Mandelman (2010). Recent papers have also modeled the bank-loans as an imperfectly competitive market (Andres and Arce, 2012).

$$c_t + m_t + \frac{R^l_{t-1} b_{t-1}}{\pi_t} = \frac{m_{t-1}}{\pi_t} + b_t + w_t n_t + \Pi_t \tag{20}$$

$$b_t \leq \eta n_t w_t. \tag{21}$$

The new set of first-order conditions is given by:

$$-\frac{u_{n,t}}{u_{c,t}} = w_t(1 + \eta \mu_{b,t}) \tag{22}$$

$$u_{c,t} = \beta R^l_t \, \mathbb{E}_t \left(\frac{u_{c,t+1}}{\pi_{t+1}} \right) + \mu_{b,t} u_{c,t} \tag{23}$$

$$u_{m,t} = u_{c,t} \left[\frac{(R^l_t - 1) + \mu_{b,t}}{R^l_t} \right]. \tag{24}$$

Note the resemblance with our original first order conditions, with the loan interest rate replacing the policy interest rate. As before, the money demand condition (equation (24)), will be important for the determination of the optimal nominal policy interest rate through its effect on the optimal nominal loan rate.

4.1 Lenders

The presence of monopolistic power implies that lenders can charge a higher nominal interest rate on loans than the nominal policy interest rate, i.e. $R^l_t > R_t$. Similarly, the market power in the deposit market implies a deposit rate that is be lower than the policy interest rate, $R^d_t < R_t$.[15] At the end of each period, banks' profits are transferred back to their owners (households) in a lump-sum fashion. Therefore, banks discount future profits using the households' stochastic discount factor, $\Lambda_{t+k|t} \equiv \beta^k u_{c,t+k}/u_{c,t}$.

We denote by $l_{i,t}$ the real value of loans extended by bank i and by $d_{i,t}$ the real value of deposits. We also let $g_{i,t}$ be the real balance (position) of bank i in the interbank market. This balance can be positive, negative, or zero, depending respectively on whether bank i is a net lender, a net borrower, or neither. Furthermore, because we focus on the market power of banks and we will be searching for the average nominal policy interest rate rather than its short run response to shocks, we abstract from stickiness of deposit interest rates or loan interest rates.

Each bank i chooses the interest rates that it offers on loans, $R^l_{i,t}$, and deposits, $R^d_{i,t}$, to maximize the expected present discounted stream of real profits. Formally, the problem of

[15] The interpretation of the gross nominal policy interest rate is similar to the Federal Funds Rate in the U.S., at which interbank lending takes place.

bank i is given by:

$$\max_{\{R^l_{i,t},R^d_{i,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \mathbb{E}_0 \, \Lambda_{t|0} \left[R^l_{i,t} l_{i,t} - R^d_{i,t} d_{i,t} - R_t g_{i,t} \right] \tag{25}$$

subject to its balance sheet , the deposit supply condition and the loan demand condition, given, respectively, by:

$$l_{i,t} = d_{i,t} + g_{i,t} \tag{26}$$

$$d_{i,t} = \left(\frac{R^d_{i,t}}{R^d_t} \right)^{\rho} d_t \tag{27}$$

$$l_{i,t} = \left(\frac{R^l_{i,t}}{R^l_t} \right)^{-\upsilon} l_t \tag{28}$$

with the parameters $\rho > 1$ and $\upsilon > 1$ being, respectively, the elasticity of substitution between two types of deposits and two types of loans. For simplicity, we assume that banks do not hold reserves. As shown in Appendix B, conditions (27) and (28) are obtained from optimization by depositors and borrowers and they are in line with Gerali et al. (2010).

In a symmetric equilibrium, in which all banks charge the same loan rate, the choice of $R^l_{i,t}$ yields:

$$R^l_t = \frac{\upsilon}{\upsilon - 1} R_t \tag{29}$$

which governs the behavior of the loan interest rate over time. This condition suggests that the loan interest rate is higher than the policy interest rate for finite values of υ, with the spread between the two rates being inversely related to the size of υ. When υ approaches infinity, which corresponds to the case of perfect competition in the loan market, we have $R^l_t = R_t$ for all t.

Similarly, we have the following condition that governs the behavior of the deposit rate:

$$R^d_t = \frac{\rho}{1 + \rho} R_t. \tag{30}$$

Therefore, the deposit interest rate is lower than the policy interest rate and it is negatively related to the degree of market power of banks in the deposit market; a lower ρ implies a lower deposit interest rate relative to the policy interest rate. The two interest rates will be equal when ρ approaches infinity.

4.2 Intermediate-Good Firms

The problem of firm (depositor) j is now:

$$\max_{\{n_{j,t},d_{j,t},P_{j,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \Omega_{t|0} \left[\frac{P_{j,t}}{P_t} y_{j,t} - w_t n_{j,t} - \frac{\varphi}{2} \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2 y_t + \frac{R^d_{i,t-1} d_{j,t-1}}{\pi_t} - d_{j,t} \right] \tag{31}$$

subject to the downward sloping demand function for its product given by equation (8).

We now get the following labor demand condition, Philips curve and the demand for deposits condition, respectively:

$$mc_t = \frac{w_t}{\alpha z_t n_t^{\alpha-1}} \tag{32}$$

$$\epsilon(1 - \alpha mc_t) = 1 - \varphi(\pi_t - 1)\pi_t + \delta\varphi\,\mathbb{E}_t \left[(\pi_{t+1} - 1)\pi_{t+1} \frac{y_{t+1}}{y_t} \right] \tag{33}$$

and

$$1 = \delta R^d_t\,\mathbb{E}_t \left(\frac{1}{\pi_{t+1}} \right). \tag{34}$$

4.3 Interest Rates and Credit Conditions

To focus on the relationship between the interest rates and the credit conditions, we start by defining $\upsilon/(\upsilon - 1) = 1 + \phi$ and letting i^l_t and i_t be, respectively, the net nominal loan rate and the net nominal policy rate. Equation (29) can then be rewritten as follows:

$$i_t = \frac{i^l_t - \phi}{1 + \phi}. \tag{35}$$

Equation (35) is one of the equilibrium conditions that characterize the solution to the optimal monetary policy problem and it implies that the solution may result in a negative nominal policy interest rate. Since ϕ measures the spread between the two interest rates, the sign of the policy rate depends on the size of the loan rate with respect to the spread. Specifically, the optimal nominal policy interest rate can potentially be negative if the optimal nominal loan interest rate falls short of the spread between the two interest rates. To see the effects of the credit constraint on the sign of the nominal policy interest rate, consider two alternative cases. First, if the economy is satiated with real money balances, then the marginal utility of cash holding is zero, $u_{m,t} = 0$. In this case, condition (6) yields the following relationship between the loan rate and the tightness of the borrowing constraint:

$$i^l_t = -\mu_{b,t} \tag{36}$$

and the nominal policy interest rate will be negative as long as the borrowing constraint is binding ($\mu_{b,t} > 0$). Broadly speaking, setting a negative nominal policy interest rate is more appropriate in tighter credit markets.

Second, if $u_{m,t} > 0$, i_t may be zero, positive, or negative. If $i_t^l \leq 0$, then clearly, due to the presence of lenders' market power, the policy interest rate will be negative. However, if $i_t^l > 0$, which is the more interesting case, then the sign of the policy interest rate will depend on the relative magnitude of i_t^l with respect to the markup, ϕ.

At the steady state, the combination of (23) and (34) gives:

$$\mu_b = \frac{\delta - \beta\left(1+\omega\right)}{\delta} \tag{37}$$

with ω being the spread between the loan rate and the deposit rate. The borrowing constraint will bind at the steady state if $\delta > \beta\left(1+\omega\right)$. In a model with perfect competition in the banking sector, equation (37) collapses to the more familiar condition, $\mu_b = (\delta - \beta)/\delta$, in which case the borrowing constraint binds whenever $\delta > \beta$, as is assumed here. Since the spread between both rates also affects the degree to which the constraint binds, the introduction of imperfect competition in the banking sector makes the tightness of the borrowing constraint less trivial. In either case, when the gap between the two discount factors increases, the borrowing constraint becomes tighter at the steady state.

In a symmetric equilibrium, the interbank lending of each bank is zero, $g_{i,t} = 0$. In addition, we have $b_{i,t} = l_{i,t}$ in the loan market. Then, in the aggregate, condition (26) implies:

$$l_t = d_t \tag{38}$$

which is the clearing condition of the intermediation sector. Intuitively, the amount of loans that banks can extend is equal to their deposits.

The resource constraint of the economy remains unchanged:

$$y_t = c_t + \frac{\varphi}{2}\left(\pi_t - 1\right)^2 y_t. \tag{39}$$

We now turn to the definitions of the private-sector equilibrium and the optimal monetary policy problem.

Definition 3 (Private-Sector Equilibrium) *Given the exogenous processes for z_t and R_t, the private-sector equilibrium is a sequence of allocations $\{c_t, d_t, l_t, m_t, mc_t, n_t, R_t^d, R_t^l, w_t, y_t, \pi_t, \mu_{b,t}\}$ that satisfy the equilibrium conditions (9), (21)-(24), (29)-(30), (32)-(34) and (38)-(39).*

Definition 4 (Optimal Policy Problem) *Given the exogenous process of z_t, the monetary authority chooses a sequence of allocations $\{c_t, d_t, l_t, m_t, mc_t, n_t, R_t, R_t^d, R_t^l, w_t, y_t, \pi_t, \mu_{b,t}\}$*

to maximize (1) subject to the equilibrium conditions (9), (21)-(24), (29)-(30), (32)-(34) and (38)-(39).

4.4 Calibration and Results

For the quantitative exercise of this section we will focus solely in the full commitment case. All parameter values are as in Table 1. In addition, the annual spread between the loan rate and the deposit rate is set to 5% (which corresponds to a 2% spread between the policy and the deposit rates and a 3% spread between the loan and the policy rates).

Table 6 shows the results. We consider three different cases: *i*) market power in both the loan and the deposit markets (the benchmark case), *ii*) market power only in the loan market, and *iii*), market power only in the deposit market. The three different scenarios point to the same conclusion. Our main qualitative result, a negative optimal policy nominal interest rate, is robust to the introduction of market power in the banking sector. The exact value of the policy interest rate, however, depends on the specific case in hand.

Table 6: Optimal Monetary Policy- Imperfect Banking Sector

Case	Variable	Mean	Std. Dev.
Benchmark	i	-1.8930	0.0060
	i^l	1.0911	0.0062
	i^d	-3.8213	0.0059
	π	-7.6113	0.0056
	y	0.3529	0.0150
Competitive Deposit Market	i	-3.7861	0.0058
	i^l	-0.8597	0.0060
	i^d	-3.7863	0.0058
	π	-7.5777	0.0056
	y	0.3536	0.0150
Competitive Loan Market	i	-1.8393	0.0059
	i^l	-1.8391	0.0059
	i^d	-3.7867	0.0058
	π	-7.5608	0.0056
	y	0.3539	0.0150

Notes: Numerical results obtained from solving the model using a second-order approximation around the non-stochastic steady state for the Ramsey problem. Moments reported are log-detrended simulations using an HP filter with a smoothing parameter of 1,600. The means of the interest rates and the inflation rate are presented in annualized percentage terms.

Consider first the case when banks have market power in both markets. In this case, the planner faces a trade-off: on the one hand, lowering the policy rate will help lowering the loan rate and thus encourage borrowing by households. On the other hand, reducing the policy rate lowers the deposit rate, which discourages deposits. The planner thus chooses a policy rate that balances both effects and, as a result, the policy rate emerges as higher (i.e. less negative) than in the model with perfect competition in the banking sector. The spread between the loan rate and the policy rate makes it possible to set a negative policy rate concurrently with a positive loan rate.

When banks have no market power in the deposit market, the policy rate and the deposit rate are equal. Relative to the benchmark case, the competition in the deposit market slightly drives up the deposit rate. The policy rate turns out to be negative and so does the loan rate, albeit not as negative as in the model with perfect competition in the banking sector. If banks have no market power in the loan market, however, the policy rate and the loan rate are, as expected, equal. In this case, the nominal policy rate will be entirely determined by the value of the nominal loan rate implied by the money demand condition. This leads to a less negative policy rate compared to the result in the model with perfect competition in the banking sector. It is also interesting that the implied deposit rate and the inflation rate are both similar across the different experiments that we consider here.

The analyses of this section, thus, indicate that the optimality of negative interest rates in this paper holds when imperfect competition in the banking sector is introduced. However, the magnitude of the nominal policy interest rate will depend upon the degree of monopolist power in this industry. If banks possess market power both in the loan market and the deposit market, the nominal policy interest rate is reduced by almost 50%. One way to see this result is by considering condition (37): the higher the spread between the loan and the deposit rate, the lower the degree of tightness of the borrowing constraint which results in the nominal policy interest rate being less negative.

5 Conclusions

Setting a negative nominal interest rate has been mentioned as an alternative policy to overcome the zero lower bound since the peak of the recent financial crisis and ensuing recession. This has raised questions about the theoretical justification in the most familiar economic theory for adopting such a policy and under what conditions this could happen.

This paper studies optimal monetary policy in an otherwise standard New Keynesian model with borrowing constraints on households and money demand. We show that a standard calibration of the model delivers a negative optimal nominal policy interest rate (around -4% annually). With borrowing constraints, sticky prices, monopolistic competition in the product market and money demand, there are three competing channels that operate on the optimal nominal interest rate: the monetary distortion (which calls for a zero nominal interest rate), the nominal distortion (which calls for a positive nominal interest rate) and the credit friction (which calls for a negative nominal interest rate). Our results reveal that, under plausible calibration of the model, the credit friction is the most dominant force, thus leading to a negative nominal policy interest rate optimally. This result essentially suggests that satiating the economy with real money balances happens only when the nominal interest rate is negative rather than zero. In other words, we show that the "Friedman Rule" is not optimal in this setup even if the premise behind it does emerge as optimal.

Our main result is robust to the introduction of monopolistic competition in the banking sector. Indeed, in a model with market power in the banking sector, the optimal nominal policy interest rate is between -2% and -4%. The exact magnitude of the nominal policy interest rate in this setup depends on whether banks have market power in the loan market, the deposit market or both.

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Appendix

A The Model with a CIA Constraint

The problem of households now is to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \tag{A.1}$$

subject to the sequence of budget constraints:

$$c_t + m_t + \frac{R_{t-1} b_{t-1}}{\pi_t} = \frac{m_{t-1}}{\pi_t} + b_t + w_t n_t + T_t + \Pi_t \tag{A.2}$$

the borrowing constraint:

$$b_t \leq \eta w_t n_t \tag{A.3}$$

and the CIA constraint:

$$c_t \leq m_t \tag{A.4}$$

The problems of firms and lenders remain the same as before.

Denoting the Lagrange multipliers on (A.2), (A.3), and (A.4) by $\lambda_{c,t}$, $\lambda_{c,t}\mu_{b,t}$, and $\lambda_{c,t}\mu_{m,t}$ respectively, we have:

$$R_t = \frac{1 - \mu_{b,t}}{1 - \mu_{m,t}} \tag{A.5}$$

Therefore,

$$i_t = \begin{cases} < 0 & \text{if } \mu_{b,t} > \mu_{m,t} \\ = 0 & \text{if } \mu_{b,t} = \mu_{m,t} \\ > 0 & \text{if } \mu_{b,t} < \mu_{m,t} \end{cases} \tag{A.6}$$

The nominal policy interest rate will be negative if the credit constraint is more tightened than the CIA constraint and it will be non-negative otherwise. As in the main text, we conclude that a negative nominal policy interest rate is more likely to be optimal when the credit constraint is relatively highly tightened.

B Deriving the Deposit Supply Condition and the Loan Demand Condition

We show here how did we derive conditions (27) and (28). Our setup is similar to Gerali et al. (2010). Depositors choose $D_{i,t}$ to:

$$Max \int_0^1 R^d_{i,t} D_{i,t}\, di \tag{B.1}$$

subject to:

$$D_t \geq \left(\int_0^1 {D_{i,t}}^{\frac{\varsigma-1}{\varsigma}}\, di \right)^{\frac{\varsigma}{\varsigma-1}} \tag{B.2}$$

where $R^d_t = \left(\int_0^1 {R^d_{i,t}}^{1-\varsigma}\, di \right)^{\frac{1}{1-\varsigma}}$ is the aggregate deposit interest rate.

Denoting the Lagrange multiplier on condition (B.2) by λ_t, the first-order condition with respect to $D_{i,t}$ reads:

$$R^d_{i,t} - \lambda_t \left(\int_0^1 {D_{i,t}}^{\frac{\varsigma-1}{\varsigma}}\, di \right)^{\frac{1}{\varsigma-1}} {D_{i,t}}^{\frac{-1}{\varsigma}} = 0 \tag{B.3}$$

or,

$$R^d_{i,t} - \lambda_t \left[\left(\int_0^1 {D_{i,t}}^{\frac{\varsigma-1}{\varsigma}}\, di \right)^{\frac{\varsigma}{\varsigma-1}} \right]^{\frac{1}{\varsigma}} {D_{i,t}}^{\frac{-1}{\varsigma}} = 0 \tag{B.4}$$

Using equation (B.2) when it holds with equality, condition (B.4) can be written as:

$$R^d_{i,t} - \lambda_t D_t^{\frac{1}{\varsigma}} {D_{i,t}}^{\frac{-1}{\varsigma}} = 0 \tag{B.5}$$

Re-arranging of condition (B.5) yields:

$$D_{i,t} = \left(\frac{R^d_{i,t}}{\lambda_t} \right)^{-\varsigma} D_t \tag{B.6}$$

Substituting condition (B.6) into the definition of the aggregate deposit rate and re-arranging yield $\lambda_t = {R_t}^d$. Substituting this result into condition (B.6) gives:

$$D_{i,t} = \left(\frac{R^d_{i,t}}{R^d_t} \right)^{-\varsigma} D_t \tag{B.7}$$

Finally, letting $\rho = -\varsigma$ and dividing both sides of condition (B.7) by the price level we have:

$$d_{i,t} = \left(\frac{R^d_{i,t}}{R^d_t}\right)^{\rho} d_t \tag{B.8}$$

which is condition (27). Notice that the value of ς in Gerali et al. (2010) is negative, which is consistent with ρ being positive in this paper.

To derive condition (28), we assume that households (borrowers) choose $L_{i,t}$ to:

$$Min \int_0^1 R^l_{i,t} L_{i,t}\, di \tag{B.9}$$

subject to:

$$L_t \geq \left(\int_0^1 {L_{i,t}}^{\frac{\upsilon-1}{\upsilon}}\, di\right)^{\frac{\upsilon}{\upsilon-1}} \tag{B.10}$$

where $R^l_t = \left(\int_0^1 {R^l_{i,t}}^{1-\upsilon}\, di\right)^{\frac{1}{1-\upsilon}}$ is the aggregate loan interest rate.

Following the same steps as above gives condition (28).